A COMPARISON OF PROBABILITY BOUNDS ANALYSIS AND SENSITIVITY ANALYSIS IN ENVIRONMENTALLY BENIGN DESIGN AND MANUFACTURE

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ABSTRACT

There is growing acceptance in the design community that two types of uncertainty exist: inherent variability and uncertainty that results from a lack of knowledge, which is variously referred to as imprecision, incertitude, irreducible uncertainty, and epistemic uncertainty. There is much less agreement on the appropriate means for representing and computing with these types of uncertainty. Probability bounds analysis (PBA) is a method that represents uncertainty using upper and lower cumulative probability distributions. These structures, called probability boxes or just p-boxes, capture both variability and imprecision. PBA includes algorithms for efficiently computing with these structures under certain conditions. This paper explores the advantages and limitations of PBA in comparison to traditional decision analysis with sensitivity analysis in the context of environmentally benign design and manufacture. The example of the selection of an oil filter involves multiple objectives and multiple uncertain parameters. These parameters are known with varying levels of uncertainty, and different assumptions about the dependencies between variables are made. As such, the example problem provides a rich context for exploring the applicability of PBA and sensitivity analysis to making engineering decisions under uncertainty. The results reveal specific advantages and limitations of both methods. The appropriate choice of an analysis depends on the exact decision scenario.

1. INTRODUCTION

Many methods have been recommended for representing uncertainty in engineering design, including precise subjective or Bayesian probability theory [1, 2], interval theory [3], imprecise probabilities [4, 5], evidence theory [6, 7], and possibility theory [8]. Previous work has compared probability theory and possibility theory [9], compared evidence theory and Bayesian theory [10], and demonstrated the potential value of using imprecise probabilities (compared to precise probabilities) in a simple example [4].

In this paper, the comparison of imprecise probabilities and traditional methods is taken a step farther. In short, the paper begins to answer when it is useful to use imprecise probabilities, whereas previous work demonstrated that is can be valuable in a simple example. Specifically, the process of using probability bounds analysis (PBA) [11] is compared to the process of traditional decision analysis (with sensitivity analysis) [12-14] in the context of environmentally benign design and manufacture (EBDM). In additional to being an important research area in its own right, EBDM provides a rich context for exploring uncertainty analysis since it involves multiple objectives and multiple sources of uncertainty.

The goal of the paper is to reveal the applicability, advantages, and limitations of the two methods for this type of problem. In
this sense, the actual outcome of the design problem—that is, the chosen design—is not as important as the process of reaching a decision. By concentrating on the process, the advantages and limitations of the method in different scenarios are explored.

In decision analysis, uncertainty is considered in a two step process. First, alternatives are compared based on nominal estimates, or base cases, of uncertain parameters. Second, the sensitivity of this comparison is explored by varying the uncertainty over a specified range. Further details of sensitivity analysis are deferred until the example in Section 4.

In PBA, the total uncertainty is incorporated into the decision and analysis in one step using a specific sub-class of imprecise probabilities called probability-boxes. This paper focuses first on demonstrating this process, and then exploring the advantages and limitations via a comparison to decision analysis. A theoretical view of PBA as a sensitivity analysis tool was presented by Ferson et al. [15], but to the authors’ knowledge, this is the first practical comparison and demonstration in engineering design and EBDM. An overview of PBA is presented in Section 1.2. For a more formal discussion of the mathematics and theoretical motivations of PBA and related methods, see for example [11, 15-19].

This section concludes by introducing the context of EBDM and then presenting a brief introduction to PBA. Section 2 introduces the example problem of selecting an oil filter design. Section 3 presents the PBA approach to the design problem. Section 4 presents the decision analysis with sensitivity analysis approach. Finally, Section 5 discusses and compares the two approaches.

1.1. Environmentally benign design and manufacture (EBDM)

Companies are increasingly concerned with the environment as consumers and legislators are realizing that a cost to society results from environmental impact. Interest is therefore growing in EBDM, a domain that examines the often competing goals of achieving economic growth and protecting the environment.

All products and processes in some way affect our environment during their entire, and often long, life span. Consequently, an evaluation of all of the loads and impacts has traditionally been addressed with life cycle assessment (LCA) methods. Researchers are starting to recognize that a key characteristic of LCA is that only very limited information and knowledge is available, resulting in large uncertainty, as summarized by Ross [20] and Björklund [21].

In general, multi-criteria evaluations that include environmental performance can be decomposed as depicted in Figure 1. Similar decompositions have been proposed [22, 23], though none are identical in form or scope to the structure presented here. Components are grouped, as indicated by dashed-lines in the figure, using Hofstetter’s concept of “spheres” of knowledge and reasoning about environmental evaluation [22].

- Technosphere: description of the product and its life cycle and an inventory of loads (e.g. emissions)
- Ecosphere: modeling of changes to the environment
- Valuesphere: modeling of the perceived seriousness or importance of changes to the environment

Any of the components in Figure 1 can be a source of uncertainty. Often some of these sources, such as form and inventory, are well characterized, while others, such as environmental damages, are much harder to characterize.

In summary, EBDM is a multi-objective problem, pursuing the often competing goals of economic growth and environmental protection, while subjected to multiple sources of uncertainty. This is a rich context in which to explore different methods of representing uncertainty and making engineering design decisions. It also offers an opportunity to contribute to the EBDM and LCA communities by demonstrating practical approaches for uncertainty management in those domains.

1.2. Probability bounds analysis (PBA)

Researchers in engineering design have recognized two types of uncertainty: imprecision and variability (see [24] for an overview). Variability is a description of naturally random behavior in a physical process or property, and it is also referred to as aleatory uncertainty, objective uncertainty, and irreducible uncertainty in the literature. Inherent variability is best represented in stochastic terms, e.g., by a probability density function.

Imprecision, sometimes called epistemic uncertainty, incertitude, reducible uncertainty, or subjective uncertainty, is due to a lack of knowledge or information. Imprecision is generally best represented in terms of intervals [3]. There are many sources of imprecision in engineering design [25],

![Figure 1: The components of an environmental analysis](image-url)
including simplified behavioral models, limited data regarding environmental factors, incompletely elicited preferences, and unknown physical relationships.

When both imprecision and variability are present, the total uncertainty can be represented using imprecise probabilities, a theory formalized by Peter Walley [5] that extends traditional probability theory by allowing for intervals or sets of probabilities. Pure imprecise probabilities present computational challenges, but by imposing some additional restrictions on imprecise probabilities, Ferson and Donald [11] have developed a similar formalism called probability bounds analysis (PBA). Although PBA is not quite as expressive as imprecise probabilities, it can still represent both variability and imprecision, and it has been shown to be useful in engineering design [4, 26].

PBA represents uncertainty using a structure called a probability-box, or p-box. Essentially, a p-box is an imprecise cumulative distribution function (CDF). Upper and lower CDF curves represent the bounds between which all possible probability distributions might lie. A p-box is less general than imprecise probabilities because the use of bounds precludes the representation of a scenario in which intermediate distributions between the bounds are excluded from the allowable set [15].

The commercially available software RAMAS Risk Calc 4.0 [27] provides one implementation of PBA by discretizing the p-box and then using algorithms developed by Williamson and Downs [28] for the operations of additional, subtraction, multiplication, and division. The methods are called dependency bounds convolutions because they result in bounds on the true probability distribution. These methods handle various dependence relationships between the uncertain quantities, including independence and unknown dependence.

A similar approach was developed independently by Berleant in [16, 29] and is implemented in the software Statool [30].

2. LCA EXAMPLE: OIL FILTER SELECTION

Around 250 million light duty oil filters are discarded (and not recycled) in the United States each year [31]. The environmental impact of these filters can be substantial, as disposable filters contain large amounts of steel, aluminum, or plastic, depending on the style of filter.

In this example, it is assumed that an automobile manufacturer wants to reduce the environmental impact of oil filters from its cars by designing a more environmentally benign filter. Naturally, the company simultaneously wants to make a profit, making this an EBDM problem. We assume that since high-price filters are less attractive to consumers than low-price filters (with all other things being equal), the manufacturer wants to minimize the total cost to the consumer of purchasing oil filters over the lifetime of the vehicle.

In the following, the example is described in detail. This extensive explanation will be useful in the subsequent sections, as a complete understanding of the example problems and assumptions is necessary to appreciate the advantages and limitations of the methods.

Naturally, some simplifications and assumptions are introduced in the problem. For example, the exact dimensions and parameters for the problem are chosen to be realistic, but are not based on hard, real-world data. Consequently, the emphasis is not on the actual decision outcome (i.e. the chosen filter), but rather on the decision and analysis process.

2.1. Types of oil filters

In this simplified model, shown in Figure 2, an oil filter is comprised of five components: housing, top cap, filter, inner support, and bottom cap. The housing, top cap, and bottom cap make up the casing, and the inner support and filter make up the cartridge. Three different types of oil filters are considered, as summarized in Table 1.

For the steel easy change (SEC) filter, the structural components are made of steel. The entire filter is designed to be replaced at once; it is simplyunscrewed from the engine and then discarded or recycled. The plastic easy change (PEC) filter is used exactly as the SEC filter, but its structural components are plastic rather than steel. Finally, the take-apart spin-on (TASO) filter has structural components made of aluminum and when the filter is replaced, only the cartridge is replaced; the casing is reused.

2.2. The design problem

The design problem used in this example is a selection between the SEC, PEC, and TASO filter types. The “best” choice depends on various factors, as shown using an influence diagram in Figure 3 (see [12] for an introduction to the usage and [32] for an overview of the history of influence diagrams). The influence diagram is constructed by first identifying the decision (i.e. filter type selection). This decision is made considering some objective, and the purpose of the influence diagram is to map what influences this objective.

2.2.1. Objective function

It was noted earlier that the manufacturer has two primary goals: to minimize environmental impact, and to keep the cost to the user low. This is shown in Figure 3 by the arrows

<table>
<thead>
<tr>
<th>Filter</th>
<th>Material</th>
<th>Discarded parts</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEC</td>
<td>Steel</td>
<td>Cartridge and Casing</td>
</tr>
<tr>
<td>PEC</td>
<td>Plastic</td>
<td>Cartridge and Casing</td>
</tr>
<tr>
<td>TASO</td>
<td>Aluminum</td>
<td>Cartridge only</td>
</tr>
</tbody>
</table>

Table 1. Types of filters
leading from “Total user cost” and “Total impact on environment” into the objective. In this example, the objective will be expressed in terms of utility, using multi-attribute utility theory [33, 34].

Specifically, the total utility is given in Equation (1), and the individual utility functions are given by Equations (2) and (3).

\[
U_{\text{total}} = U_{\text{cost}} + w \cdot U_{\text{impact}} \\
U_{\text{cost}} = -\text{cost} \\
U_{\text{impact}} = -\text{ecoscore}
\]

The quantity \text{ecoscore} is explained in the next subsection. As defined, all utilities will be negative, but they remain relevant for comparisons. This simple utility function is chosen to highlight the influence of uncertainty representations on the decision process. The elicitation of exact multi-attribute utilities is challenging, so the weight parameter \( w \) is assumed to be imprecise, confined only to the interval \( w = [0.5, 2] \), with the best guess at \( w = 1 \). In practice, the functional form of the utility functions could also be uncertain, a scenario which is not addressed in this paper.

### 2.2.2. Ecological impact calculation

It is assumed that the primary environmental impact of an oil filter arises due to the construction, transportation, and disposal of the casing and cartridge. These components are constructed of materials such as steel, aluminum, and plastics and present in large quantities. Other substances, such as the cellulose filter element and oil residue, are present in much smaller quantities and are generally equivalent in all three types of filters.

The Eco-indicator 95 is an impact assessment method for life-cycle analysis in which particular scores (\text{ecoscores}), measured in eco-points, are assigned to specific materials and processes. There is also an updated Eco-indicator 99 available [35], but for illustrative purposes the old database and methodology is sufficient. Since these scores are given for specific materials as points per mass, we will refer to them as Eco-indicator rates, or simply \text{ecorates} in this paper. The impact of a filter of material \( m \) is computed using Equation (4).

\[
\text{impact}_m = \text{ecoscore}_m = \text{mass}_m \cdot \text{ecorate}_m
\]

For a particular material, the ecorate not only captures its environmental effects and damages, but also sets a value on these damages relative to other damages. As such, it combines the ecosphere and valuesphere components of Figure 1. This allows for tradeoffs between different materials and processes with different inherent environmental impacts. These value tradeoffs are fixed within the Eco-indicator model, but in practice, not every society or decision maker will agree with these tradeoffs. Consequently, the valuesphere is a source of uncertainty in environmental life-cycle assessment. The effects and damages are uncertain due to the complexity and uncertainty of modeling ecosystems.

In Figure 1 the ecosphere is independent of the technosphere because the ecorates are independent of what effects are present; they are pre-tabulated for all materials. In this problem, the technosphere effects, or inventory, is given by the total mass of material used over a vehicle’s lifetime, and are thus incorporated into the problem in Equation (4).

The total mass depends on the number of filters \( F \) used, as shown in Table 2. This number varies because not every vehicle is in service for the same number of miles, and car owners change the filters with difference frequencies. Both \( L \) and \( f \) are thus uncertain. The number of filters used over a vehicle’s lifetime is given by Equation (5).

\[
F = \frac{L}{f}
\]
2.2.3. Total user cost calculation

The total cost is also function of the number of filters $F$ used over the vehicles lifetime and the price of filter parts. This function depends on the type of filter, since different parts are used for different filter types. The functions and prices are summarized in Table 2. In general, filter price is uncertain due to unknown market fluctuations and inflation. For simplicity in this example, the costs are assumed constant.

<table>
<thead>
<tr>
<th>Filter</th>
<th>Total mass of $F$ filters (kg)</th>
<th>Total cost of $F$ filters</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEC</td>
<td>$(mass_{casing} + mass_{cartridge}) \cdot F$</td>
<td>$5 \cdot F$</td>
</tr>
<tr>
<td>TASO</td>
<td>$mass_{casing} + (mass_{cartridge}) \cdot F$</td>
<td>$15 + 5 \cdot F$</td>
</tr>
<tr>
<td>PEC</td>
<td>$(mass_{casing} + mass_{cartridge}) \cdot F$</td>
<td>$8.5 \cdot F$</td>
</tr>
</tbody>
</table>

Table 2. Total environmental impact and cost functions

2.3. Assumptions on available information

The assumed uncertainties are summarized in Table 3. The ecological impact per unit mass is known to be within a stated interval for each material. Interval data is assumed because the uncertainty in the numbers is not probabilistic but rather arises from modeling errors and assumptions about the ecosphere and valuesphere. The stated intervals represent 50% to 200% of the nominal values calculated using the Eco-indicator 95 analysis (6.2, 21.2, and 9.4 millipoints per kilogram for SEC, TASO, and PEC filters respectively).

Imprecise probabilities are used to represent the uncertainty in the vehicle lifetime and filter change frequencies. The variability arises because the population of vehicle owners contains a variety of individuals, each who has his or her own behavior, but who collectively appear random. For illustration, only one parameter of the distributions is assumed to be known imprecisely, but the methods immediately generalize to multiple uncertain parameters. Several reasons for imprecisely known probabilities include:

- Limited relevant historical data for a new product
- Incomplete characterization of market segment for a new product, e.g. imprecisely known customer population
- Changing behavior due to outside influences, e.g. laws

It is assumed that $L$ and $f$ are both independent of all three eco-rates, and that the weighting $w$ is independent of all other parameters. However, the dependency between $L$ and $f$ is unknown, as are all dependencies between all ecorates.

Why is independence not assumed? $L$ and $f$ are both related to user behavior. It is conceivable that a user who intends to keep a car a long time will change the filter at a higher rate than someone who keeps a car a short time, since the long-time owner would have a greater interest in keeping the engine in good condition. In such a scenario, $L$ and $f$ are correlated. However, this dependence is not known exactly and may not even exist at all, so it makes sense to assume an unknown dependency. A similar argument can be made between the ecorates; they could be independent since they relate to different material and potentially different environmental effects and damages. However, they could also be correlated if, for example, they share an effect in the ecosphere.

A traditional statistical approach would require perfect knowledge of all joint probabilities, information that is rarely known. Consequently, independence between uncertain parameters is commonly assumed, an assumption that is often unjustifiable given available information. The ability of PBA to handle unknown dependencies, and therefore compute the possible range of results with just the marginal distributions as inputs, is a major advantage over traditional methods.

3. OIL FILTER SELECTION USING PBA

The objective function in this example, Equation (1), has two components: cost and environmental impact. However, the cost calculation is much simpler and facilitates a clearer demonstration of the PBA process. Consequently, we first consider the single objective cost to illustrate some properties of PBA, and then return to the multi-attribute EBDM problem later. The commercially available Risk Calc software is used for all calculations in this section.

3.1. Total cost calculation

According to Figure 3, total cost is a function of the number of filter changes $F$, which according to Equation (5) is strictly the quotient of $L$ divided by $f$. This calculation is evaluated in Risk Calc by first defining the variables $L$ and $f$ according to Table 3, which results in the p-boxes shown in Figure 4 and Figure 5 respectively. The result of the quotient is then calculated directly using Equation (5) and results in the p-box for the total number of filter changes $F$ shown in Figure 6.

<table>
<thead>
<tr>
<th>Uncertain parameter</th>
<th>Assumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle lifetime $L$ (miles)</td>
<td>$Gamma(\alpha_1, 2)$, $\alpha_1 = [40000, 60000]$</td>
</tr>
<tr>
<td>Filter change frequency $f$ (miles/filter)</td>
<td>$3000 + Weibull(\alpha_2, 5)$, $\alpha_2 = [3000, 5000]$</td>
</tr>
<tr>
<td>Utility weighting $w$</td>
<td>$[0.5, 2.0]$</td>
</tr>
</tbody>
</table>

Table 3. Assumptions about uncertainty
To calculate the total cost, the p-box is merely scaled and shifted according to the equations in Table 2. The result for the SEC filter is shown in Figure 7. The p-box in Figure 7 captures the set of cumulative probability distributions for total cost of the SEC filter that are consistent with the available data. The procedure can be repeated for the TASO and PEC filters using the appropriate equations. The next step is to compare the results for the three alternatives.

### 3.2. Comparing p-boxes

In traditional statistical decision theory, precise probability distributions are assumed, and a decision maker compares alternatives by taking mathematical expectation over the distributions and selects the alternative with the lowest expected cost. For a p-box, a similar calculation can be made, but since the p-box consists of multiple distributions, multiple expected costs result. Together they form an interval of expected cost for each alternative.

The intervals of expected cost can be calculated using the bounding distributions of the p-box. The resultant intervals of expected cost are shown in Figure 8. When two intervals of expected cost overlap, as in Figure 8, the choice of the best alternative is indeterminate. Depending on where in the two intervals the true values lie, either one could be the best choice, but the available information does not present a rational way to determine which really is. In some cases, this indeterminacy can be resolved, as follows.

### 3.3. Resolving indeterminacy

When the basic analysis results in indeterminacy of choice, the decision maker has several options, including:

1. Collect additional information
2. Make an arbitrary, satisficing, or robust decision
3. Base the decision on additional criteria
4. Include information in the decision that is not captured in the intervals, such as shared uncertainty

The first option is straightforward—collect information until the indeterminacy is eliminated or until it is no longer valuable to reduce it. As an example of the second, assume that the decision maker is convinced that an oil filter with a lifetime cost to the average user of greater than $130 is unacceptable. In that case, any filter whose interval is above or includes this value is unacceptable. In a satisficing approach, any alternative that meets this constraint is acceptable. In this example, only the SEC filter meets this criterion with certainty, so it would be chosen. According to the intervals in Figure 8, it is possible that the PEC and TASO filters perform better than the SEC filter, but the SEC filter is guaranteed to meet the minimum requirement, while the other are not.

One possible arbitrary policy is to choose the alternative with the lowest lower bound (a so-called mini-min policy), which suggests risk-taking or optimistic behavior. An opposite policy is to choose the alternative with the lowest upper bound (a so-called mini-max policy), which suggests risk-aversion or pessimism. In this example, all three policies result in the same choice, but this will not always be the case.

The third option is to include additional criteria into the decision. In this example, the decision maker’s actual goal is to minimize both the cost and environmental impact of the filter, so the impact information could be included in the decision with the hope of resolving the indeterminacy.

The fourth option for resolving indeterminacy is to include additional information. This is possible because any mathematical representation of information is an abstraction; generally, some information is lost when it is forced into a particular formal representation. It is not always economical to use the most expressive formalism, because the complexity of a formalism increases the computational and informational costs.

![Figure 8. Intervals of expected cost](image-url)
In the filter example, one possibility source of additional information to include is shared uncertainty [25].

3.4. Consideration of shared uncertainty

Accounting for shared uncertainty when handling imprecision is similar in goal to using joint probability distributions or correlations when handling precise probabilities; the goal is to explicitly recognize events that tend to occur together (or separately). In this example, the number of filters \( F \) used is assumed to be independent of the filter design; the choice of filter has no impact on the number of times it will be replaced. Consequently, the imprecision in \( F \) is the same, or shared, for all three alternatives.

In the preceding analysis, this commonality was not considered. For example, the upper bounds on the intervals shown in Figure 8 actually all correspond to the distribution of \( F \) with the largest mean value. Consequently, it only makes sense to compare the upper bound of the SEC cost to the upper bounds for TASO and PEC rather than the entire intervals. One way to account for such shared uncertainty is to compare the differences between the alternatives at every realization of \( F \), a so-called maximality comparison [5, 25] that is similar in motivation for paired statistical testing [36]. The need for considering these differences explicitly is further explained in the following section, but first the results are presented.

The resulting cost difference intervals are shown in Figure 9. In this figure, the intervals are not compared directly to each other, but rather each interval is compared to zero. Starting at the left, interval for the expected difference in cost between the SEC filter and the PEC filter (\( \text{cost}_{\text{SEC}} - \text{cost}_{\text{PEC}} \)) lies entirely below zero, so (since the decision maker wants to minimize cost) the SEC filter is preferred to the PEC filter (based on expected cost alone). Similarly, the interval for the difference between SEC and TASO is entirely below zero, meaning SEC is preferred to TASO. Consequently, it becomes clear that SEC is the best alternative in terms of minimizing expected cost, a conclusion that could not be drawn from Figure 8.

3.5. Interval arithmetic and repeated variables

In the previous section, the importance of considering shared uncertainty when comparing alternatives was illustrated. The explicit consideration of shared uncertainty in PBA is necessitated partly by a fundamental limitation of interval arithmetic. In general, deterministic interval arithmetic algorithms are rigorous but not best possible. In interval arithmetic (and by extension in probability bounds analysis), upper and lower bounds are best possible if the upper bound is a low as possible and the lower bound is as high as possible without conflicting with the true state of uncertainty. Bounds are rigorous as long of the best possible bounds are included between them.

Repeated variables in an expression usually lead to bounds that are not best possible. This means that in interval arithmetic it is not always true that \( A \cdot B + A \cdot C = A \cdot (B + C) \) [27]. This is a definite limitation of interval methods and PBA. However, we emphasize that the methods are rigorous, meaning that the true interval \( A \cdot (B + C) \) is always contained in the calculated interval \( A \cdot B + A \cdot C \), so the calculated bounds are true, but not best possible. In some cases, the repeated variables can be eliminated by rearranging or factoring the equations, but in general, there is no way to avoid this problem for all calculations. In this example, the formulae are such that repeated variables cannot be eliminated. Consequently, the results may not be best possible.

Since the bounds are rigorous, an alternative will never appear to be the most preferred when in fact it is not the most preferred. However, if the bounds are much larger than the best possible bounds (a situation referred to as overly conservative), then there may appear to be significant indeterminacy when in fact the real problem may involved none.

For example, the p-boxes for differences in utility between the SEC filter and TASO filter both considering and not considering the shared parameter is shown in Figure 10. The true p-box (solid line) is actually much more restrictive than the one (dotted line) calculated ignoring the presence of repeated variables. An overly conservative p-box will lead to intervals of expected utility that are larger than necessary, possibly causing indeterminacy in the decision.

3.6. Multiple objective analysis and selection

In the previous section, various properties of PBA were demonstrated and discussed in the context of cost minimization. Attention is now returned to the actual multiattribute example problem. Using Risk Calc, all of the uncertainties shown in Table 3 can be propagated through the problem to evaluate overall utility. The resulting intervals for the comparisons of design alternatives are shown in Figure 11. Based on these results, no decision can be made because all.
three intervals contain zero. Such indeterminacy can be expected if the uncertainty is large, because large uncertainty implies a lack of information for determining the best alternative.

The indeterminacy implies that more information or a different decision policy is needed, such as discussed in Section 3.3. However, because the PBA calculations are not always best possible, the indeterminacy shown in Figure 11 may not actually exist. This point is revisited in the discussion section.

4. OIL FILTER SELECTION WITH DECISION ANALYSIS AND SENSITIVITY ANALYSIS

Decision analysis is a discipline that studies procedures, tools, and frameworks for transforming problems that are difficult to understand, solve, or explain into problems that are more readily understood and solved [12, 13]. This section presents a basic decision analysis, including sensitivity analysis, approach to selecting an oil filter.

4.1. Basic decision analysis

Clemen (page 6) describes the decision-analysis process with the following steps:

1. Identify the situation and understand objectives
2. Identify alternatives
3. Decompose and model the problem structure, uncertainties, and preferences
4. Choose the best alternative
5. Perform sensitivity analysis

Steps 1-3 have been completed in Section 2. The resulting data is the same as in Table 3, but the uncertainties will be treated differently. Traditional decision analysis does not explicitly recognize imprecision, so the analysis in step 4 is performed using best-guess, base values for all of the imprecise parameters, as shown in Table 4. Notice that this data is consistent with the data in Table 3.

The expected value of the objective function for each design alternative can be calculated using the relationships described earlier. The results are shown in Table 5. Assuming the base values for all parameters, the TASO filter has the highest expected utility and is therefore the best alternative (recall that as the utility function was defined in Equations (1)-(3), all of the utilities were guaranteed to be negative). The next step in decision-analysis is to perform a sensitivity analysis on selection of the TASO filter, discussed in the next subsection.

4.2. Sensitivity analysis

The use of the base values in the previous analysis ignores the knowledge about the uncertainties that was described in Table 3. In a sensitivity analysis, a decision maker asks, “How might this neglected uncertainty affect the decision?”

The phrase sensitivity analysis has been used to refer to various procedures in engineering design. One convenient way of performing a sensitivity analysis for a selection decision is to evaluate the sensitivity of the decision outcome graphically using tornado diagrams [12, 13, 37], such as shown in Figure 12 for the SEC filter. A tornado diagram allows a decision maker to perform a one-way sensitivity analysis--that is, to explore the effects of uncertain parameters one at a time. The

<table>
<thead>
<tr>
<th>Uncertain parameter</th>
<th>Assumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle lifetime $L$ (miles)</td>
<td>$Gamma(50000,2)$</td>
</tr>
<tr>
<td>Filter change frequency $f$ (miles/filter)</td>
<td>$3000 + Weibull(4000,5)$</td>
</tr>
<tr>
<td>Eco-impact rate (millipoints per kg)</td>
<td>Steel 6.2, Aluminum 21.2, Plastic 9.4</td>
</tr>
<tr>
<td>Utility weighting $w$</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 4. Base values for imprecise parameters
The first step in constructing a tornado diagram is to define upper and lower bounds for the uncertain parameters, such as those shown in Table 3. The next step is to take one parameter and vary it from its lower limit to its upper limit with all other parameters held constant at their base values. This results in an interval of values for the objective function. Finally, this is repeated for each parameter. The base values are marked with dashed lines.

The tornado diagram is useful for determining which parameters have a large potential to affect the decision, given the stated uncertainty bounds. This is often done by comparing the performance to a reference line that represents the performance of a precisely characterized alternative. If a bar crosses this reference line, then the decision is (one-way) sensitive to the corresponding parameter. This information can then guide information collection or modeling decisions.

In the oil filter example, all three alternatives involve uncertainty, so there is no constant reference line for comparison. Instead, the difference in performance between alternatives can be compared, such as shown in Figure 13 for each of the necessary pairings. For pairwise comparisons, the reference line becomes zero. If a bar crosses this reference line, then the decision is (one-way) sensitive to the corresponding parameter. This information can then guide information collection or modeling decisions.

In the discussion section, PBA and decision analysis are directly compared and contrasted in four areas: veracity, acuity, complexity, and flexibility.

5.1. Veracity of the analysis

The example problem revealed that a one-way sensitivity analysis can lead to the conclusion that the decision is insensitive to the uncertainty, while the PBA analysis of the same problem can indicate that the solution is very sensitive to the uncertainty. An obvious question to ask is which one gives the right result? Unfortunately, this question has no straightforward answer.

Due to repeated variables in the interval calculations, PBA gives bounds that may be overly conservative (too broad). On the other hand, one-way sensitivity analysis ignores dependencies and higher order interactions and can lead to results that are non-rigorous, i.e., that are inconsistent with the truth. If the selection problem is recast as a hypothesis testing, the types of errors made with the PBA and sensitivity analyses can be discussed in standard statistical terms [36].

Consider the null hypothesis that either the PEC or the SEC filter is the best choice. The alternative hypothesis is that the TASO filter is the best. A sensitivity analysis may
underestimate the true uncertainty and indicate that there is enough evidence to reject the null hypothesis in favor of the alternative when there really is not sufficient evidence to do so. In this situation, the null hypothesis could be rejected when it is true, a Type I error.

Conversely, PBA may overestimate the uncertainty and lead to the failure to reject the null hypothesis when it is false, a Type II error. A Type II error is an error in the sense that an opportunity to make a decision is lost; the null hypothesis could have been rejected, but was not. Consequently, a decision maker may waste resources or make an arbitrary decision trying to reduce indeterminacy that does not exist in the actual problem.

Which is preferable, a Type I or Type II error? A Type II error may be preferable in high-risk applications; when the cost of failure is high, one is often more willing to be conservative and spend additional resources to reduce uncertainty further. In other applications, the cost of delaying a decision or collecting more information may exceed any potential benefit from waiting. There is no general answer; the analyst must assess the situation and make his or her own choice.

5.2. Acuity of analysis

One goal of sensitivity analysis is often to determine what additional information could best improve the decision. To this end, the breakout of uncertainty and sensitivity into individual parameters in one-way sensitivity analysis is an advantage. By considering each parameter independently, the decision maker gains insight into the sensitivity of the decision to each parameter.

PBA considers all uncertainties simultaneously, accounting for all interactions and dependencies, but it does not identify the individually important sources of the sensitivity. If the PBA analysis determines that the decision is not sensitive to the overall uncertainty, this is not a problem. However, in a case like Figure 11 in which there is indeterminacy, a decision maker would benefit from guidance into resolution of the indeterminacy.

For example, based on the sensitivity analysis in Figure 13, there seems to be no need to increase knowledge about vehicle lifetime. On the other hand, the difference between SEC and TASO filter is very sensitive to the ercorate of steel, though not enough (as a one-way effect) to change the decision. The sensitivity analysis suggests that any additional information collection focus on characterizing the environmental effects. The basic PBA analysis does not provide this insight.

Ferson et al. [15] have suggested using a meta-sensitivity analysis with PBA. Traditional sensitivity starts with the base values and systematically varies one parameter at a time to its extremes. Since PBA can capture all of the uncertainty at once, the opposite approach can be taken. The “base” case becomes the results with all of the uncertainty considered, and then each uncertain parameter is “pinched” down to a zero-variance interval, a precise probability, or even a point value and the reduction of uncertainty in the result is observed.

The one-way nature of the meta-sensitivity analysis is actually beneficial. When deciding whether to spend resources collecting information about a particular parameter, the decision maker is specifically interested in effect on the overall uncertainty of reducing the uncertainty in that particular source. This type of information is not available in traditional sensitivity analysis. It thus appears that the most accurate identification of sensitivity is achieved by a hybrid approach. Additional research into such approaches is underway.

5.3. Complexity of analysis

A one-way sensitivity analysis is computationally inexpensive. In addition to the solution with the base values, each uncertain parameter requires just two calculations—one for the upper bound and one for the lower bound. Each of these calculations may involve one Monte Carlo loop to calculate expected values, although in many cases this is unnecessary. Either way, the computational complexity is generally less than with PBA.

The advantage quickly switches to PBA if two-way (or higher) sensitivity analysis is performed, especially when nested Monte Carlo loops are used. PBA computations using dependency bounds convolutions [28] are generally much faster than traditional sensitivity analysis [15, 40]. However, dependency bounds convolutions require an open, operationally defined model (e.g. algebraic) of the problem. Consequently, they cannot be used to analyze models such a differential equations, simulations, and finite element analysis. Current research establishes methods for propagating p-boxes through "black box" models, or models with unknown or complicated structure [17], and a comparison of these methods of PBA with sensitivity analysis is an area of future work.

5.4. Flexibility of the analysis

Another advantage of PBA is its inherent flexibility. We have already discussed PBA’s flexibility in terms of assumptions of independence or unknown dependence within the context of the EBDM example. Recent algorithms also handle the pairwise dependencies of maximal or minimal correlation, correlation, linear relationship and correlation within a specified interval, and signed (positive or negative) correlation [15].

In this paper, the flexibility with regard to imprecisely known distribution parameters was demonstrated, but PBA can also handle cases of unknown distribution type [19]. For example, a p-box can be constructed and propagated with only knowledge
of the mean and variance; no assumption of distribution type (e.g. normal, lognormal, gamma, or Weibull) is necessary. This would be useful in the filter selection if, for example, the decision maker had an estimate of the mean and variance of filter change frequencies, but no theoretical or empirical evidence about the distribution family.

Sensitivity analysis ignores dependencies and higher order interactions, and it requires a known distribution type. Consequently, the types of problems that can be accurately explored with sensitivity analysis are more limited than PBA.

6. CONCLUSIONS AND SUMMARY

The example decision of oil filter selection in this paper has multiple objectives and multiple sources of different types of uncertainty. The selection problem is approached with two methods: probability bounds analysis (PBA) and traditional decision analysis with sensitivity analysis. The applicability of PBA to the example problem is illustrated and discussed in detail.

Sensitivity analysis can identify important sources of uncertainty, but it can also lead to an incorrect selection because it neglects dependencies and interactions. PBA can compute with unknown distributions types, unknown dependencies, and uncertain parameters. It also provides a rigorous and global sensitivity analysis. However, PBA may yield overly conservative results (bounds that are bigger than necessary), and it is computationally more complex than simple one-way sensitivity analysis. PBA also requires meta-analysis to identify the important sources of uncertainty, but this meta-analysis is more valuable than standard sensitivity analysis.

In short, both PBA and sensitivity analysis are useful in engineering. This conclusion identifies PBA as a valid option in engineering problems, an option that while having its own limitations, clearly reveals more information in some scenarios than traditional decision analysis. As such, PBA should receive continued attention and development in the design community.

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