

Robust Materials Design of Blast Resistant Panels

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Blast resistant panels (BRPs) are panels that are designed to dissipate blast energy through allowable plastic deformation in order to protect against damage from an incoming explosion. The blast resistant panels considered in this paper are metal sandwich structures that consist of two outer face sheets and a honeycomb core. In practice, materials for BRPs are selected from the finite set of available materials; however, selection of materials places bounds on the performance of the panel. Alternatively, by designing the BRP material concurrently with the product, the material properties could be tailored such that the weight of the panel is minimized while meeting deflection constraints.

As a step towards this goal, the properties of the BRP material are defined as uncertain design variables rather than static design parameters in order to maintain some design freedom for a subsequent material design or selection problem. If a material does not exist that satisfies the ranges of material properties identified through the design process, a material design would be needed to specify the necessary component materials and processing path to realize the desired material properties. To maintain the performance of the BRP in spite of the uncertainty in the material properties, the BRP should be robust to variation in the material property design variables. Moreover, because blasts of different impulse amplitudes and duration are expected, the design of the BRP should also be robust to variation in blast pulse amplitude and duration.

In this paper, we demonstrate the design of a blast resistant panel that is robust to variation in material properties as well as variation of impulse amplitude and duration. By considering the material properties of the panel to be uncertain design variables, the panel design obtained through robust design methods provides ranges of acceptable material properties for the framing of a subsequent material design or selection problem. Additionally, the impact of the mass per unit area constraint is analyzed.

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Nomenclature

B	in-plane spacing of webs of square honeycomb core
BRP	blast resistant panel
d_i^+, d_i^-	deviation variables for goals in cDSP
cDSP	compromise decision support problem
h_c, h_f, h_b	thickness of core webs and face sheets, respectively
$\delta, \Delta\delta$	deflection of back face, variance in deflection of back face
$\bar{\epsilon}_c$	average crushing strain of core
$g_\delta, \Delta g_\delta$	deflection constraint function, variance in deflection constraint
$g_M, \Delta g_M$	mass constraint function, variance in mass constraint
$g_{SH1}, \Delta g_{SH1}$	front face shear constraint 1, variance in front face shear constraint 1
$g_{SH2}, \Delta g_{SH2}$	front face shear constraint 2, variance in front face shear constraint 2
Γ_{SH}	front face shear constraint 1 value
H, \bar{H}	thickness of undeformed and deformed core, respectively
KE_I, KE_{II}	kinetic energy per unit area in panel after stages 1 and 2, respectively
L	half-width of plate
λ_c, λ_s	factors governing strength of core in crush and stretch
$M, \Delta M$	total mass/area of plate, variance in total mass/area
μ_p, Δ_p	mean and standard deviation of peak pressure
μ_b, Δ_t	mean and standard deviation of characteristic pulse time
p_0	peak pressure of free-field pulse
ρ_f, ρ_b, ρ_c	density of front face sheet material, back face sheet material, and core base material
$\Delta\rho$	variance in density
R_c	relative density of core
$\sigma_{Y,f}, \sigma_{Y,b}, \sigma_{Y,c}$	yield strength of front face sheet material, back face sheet material, and core base material
$\Delta\sigma$	variance in yield strength
t_0	characteristic time of incident pressure pulse
$T_\delta, T_{\Delta\delta}$	targets for deflection and variance of deflection goals
W_1, W_2	weighting factors in deviation function
W_{III}^p	plastic work per unit area dissipated in stage three

I. Frame of Reference

Typically in product design processes, materials are *selected* based on product performance goals. However, in many product designs, the selection of materials places bounds on product performance capabilities, and performance goals may not be met due to limitations in the finite set of available materials. We therefore suggest that developing the capability to design new materials or tailor existing materials to meet specific product performance goals is warranted.

Materials *design* refers to the process of tailoring material properties to meet specific performance requirements. Successful materials design relies on complex multiscale material simulations to predict the performance of the designed material, and the development of these complex material models is not trivial. Hence, it is important for designers to be able to investigate the need for new materials in the context of a product design process. In an effort to understand the need for materials design, we investigate design example problems. The example problem that we address in this paper is the design of a blast resistant panel (BRP). Our intention in this paper is to use BRP design as an example in exploring robust materials design methods.

The blast resistant panels considered in this paper are sandwich structures that consist of two outer face sheets bonded by a core structure. BRPs are designed to absorb large amounts of energy per unit mass compared to solid plates. In one application of this design, BRPs can be attached on the outside of military vehicles to protect them from explosions. Examples of blast resistant panels are shown in Figure 1.

Although work has been done in the design, optimization, and analysis of blast resistant panels, there is little information in the publishing domain on designing blast resistant panels that are robust to changes in impulse amplitude and duration. Owing to the fact that blasts of different impulse amplitudes and duration can be expected, the design of the BRP should be robust to this variation. The directionality of the blast, which may give rise to spatial gradients of pressure along the BRP surface, is not considered here. In addition, to maintain design freedom in the materials of the BRP, the BRP should be robust to uncertainty in the material-related design variables. In this

paper, we design a blast resistant panel that is robust to uncertainty of impulse amplitude and duration, as well as to uncertainty in the material.

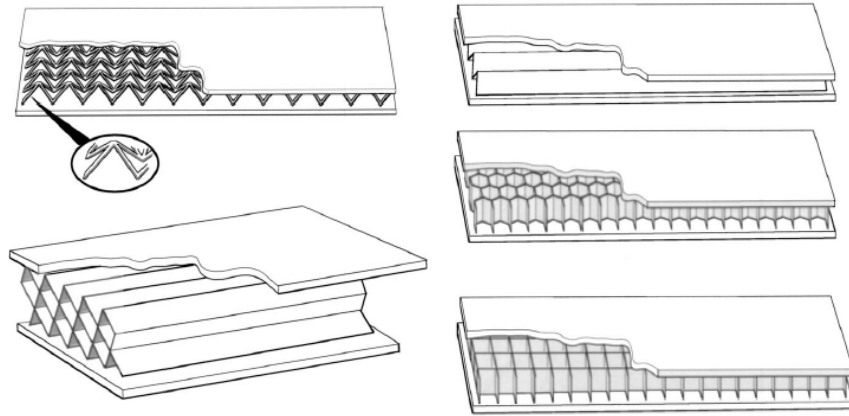


Figure 1. Sample blast resistant panels ¹

II. Utilizing the Compromise Decision Support Problem for Robust Design

Robust design is the practice of improving the quality of products by reducing sensitivity to uncertainty in noise factors, control factors (design variables), and models. When product designs are robust, performance levels remain stable despite the presence of noise factors^{2, 3}. In typical robust design problems, the variance of the performance objective function is minimized with respect to environmental conditions, loading conditions, material properties, and other noise factors⁴. The variance of the response is minimized while also maximizing the response, minimizing the response or bringing the response to a target. The robust solution may be inferior to the optimum solution in the absence of variation; however, the robust solution produces predictably satisfactory results in the presence of variation. In particular, problems involving the use of processing and fabrication of materials often exhibit substantial variability in process route, material composition, and material behavior, so robust solutions are often favored.

The compromise Decision Support Problem (cDSP) is the mathematical construct through which the conflicting goals in robust design are modeled. The cDSP is a general framework for solving multi-objective, non-linear, optimization problems⁵. Mathematically, the cDSP is a multi-objective decision model which is a hybrid formulation based on Mathematical Programming and Goal Programming⁶. It is used to determine the values of the design variables, which satisfy a set of constraints and bounds and achieve as closely as possible a set of conflicting goals. The word formulation of the cDSP is as follows:

- Given:* A feasible alternative, assumptions, parameter values and goals
- Find:* Values of design and deviation variables
- Satisfy:* System constraints, system goals, and bounds on variables
- Minimize:* Deviation variable that measures distance between goal targets and design points

The system descriptors (system and deviation variables, system constraints, system goals, bounds, and the deviation function) are described in detail by Mistree, Hughes and Bras⁶ and are not repeated here. The mathematical formulation of the cDSP is shown in Figure 2. The word and mathematical formulations of the cDSP are used as templates as we proceed with the problem formulation in Section 3.

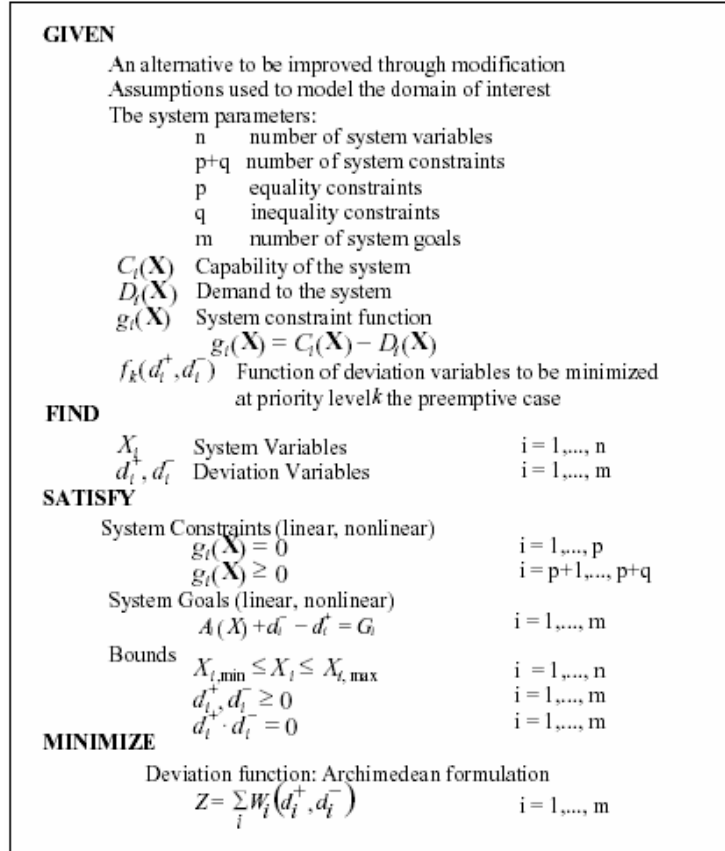


Figure 2. Mathematical Formulation of the Compromise Decision Support Problem⁶

III. Blast Resistant Panel Problem Formulation

A. Blast Resistant Panel Design

Optimization and deformation analysis of BRPs (also sandwich panels) with internal truss structures have been conducted by many researchers. Information on the design, manufacture, and testing of sandwich structures is detailed in the work of Zenkert^{7, 8}. The superior performance of blast resistant panels with internal truss structures has been verified by Xue and Hutchinson⁹. Additionally, they detailed many theoretical relationships between impulse loading and BRP deformation. The effect of topology in the internal matrix of BRPs is discussed by Xue and Hutchinson¹⁰, these effects are verified using FEA.

The deformation of sandwich plates under impulse loading is divided into three time periods according to the three-stage deformation theory developed by Fleck and Deshpande¹. Fleck and Deshpande analyze the stages of deformation, and propose equations relating impulse loading to deformation. In subsequent work, Xue and Hutchinson adapted the three-stage deformation theory proposed by Fleck and Deshpande and applied it to the optimization of blast resistant panels¹¹. The authors modeled and explored the material behavior in each stage of deformation verified the theory they proposed using FEA.

Our goal in the BRP design example is to design a blast resistant panel for minimum deflection under an uncertain blast load and with uncertain material properties. The design parameters include the thickness of each layer, the material properties for each layer and the geometric parameters relating to the topology of the honeycomb core. The panel must not exceed a maximum deflection and a maximum mass per unit area, and must not rupture or collapse under the blast loading. The panel is assumed to be clamped on all edges. The word formulation of the BRP cDSP is shown in Figure 3.

<p>Given:</p> <ul style="list-style-type: none"> • A impulse load defined by peak pressure, p_0, and characteristic time, t_0 • Mean and variance of noise factors (t_0, p_0) • Variance of uncertain control factors (ρ, σ_Y) • Model for the deflection of the panel • Constants pertaining to the geometry of the core • Area = 1 m² <p>Find:</p> <ul style="list-style-type: none"> • Material properties: $\sigma_{Y,f}, \rho_f, \sigma_{Y,b}, \rho_b, \sigma_{Y,c}, \rho_c$ • Core geometry: h_c, B • Height of each layer: h_f, h_b, H • Value of deviation variables d_i^+, d_i^- ($i = 1,2$) <p>Satisfy:</p> <p><i>Constraints</i></p> <ul style="list-style-type: none"> • Mass/area of BRP must not exceed 100 kg/m³ • Deflection must not exceed 10% of span for specified boundary conditions • Relative Density must be greater than 0.07 to avoid buckling • Front face shear-off constraints • Uncertain control variables must stay within bounds <p><i>Goals</i></p> <ul style="list-style-type: none"> • Minimize deflection • Minimize variance of deflection <p><i>Bounds</i></p> <ul style="list-style-type: none"> • Upper and lower bounds for system variables <p>Minimize:</p> <ul style="list-style-type: none"> • Deviation functions <ul style="list-style-type: none"> ○ Scenario 1: Minimize deflection only ○ Scenario 2: Minimize variance of deflection only ○ Scenario 3: Equal priority

Figure 3. Word Formulation of BRP cDSP

The blast resistant panel consists of a front face sheet, cellular core, and back face sheet as shown in Figure 4. The front face sheet receives the initial pressure loading from the blast. The topology of the core is designed to dissipate a majority of the impulse energy in crushing. The back face sheet provides additional protection from the blast as well as a means to confine the core collapse and absorb energy in stretching. A square honeycomb structure is chosen for the cell shape of the core material because the square shape resists stretching of the back face sheet more so than triangular-, hexagonal- and chiral-shaped cores.

In Table 1, we present the design variables and noise factors. For the BRP design there are eleven design variables which consist of six material property variables and five geometric variables. The material for each layer of the panel is assumed to have an elastic, perfectly-plastic stress-strain relationship, and the material is assumed to be defined by independent yield strength and density variables. The bounds for these material property design variables are determined from the ranges of properties for engineering metals. The uncertainty in these design variables was chosen to be 5% of the bounds. The five geometric design variables are used to define the height of each of the layers as well as the cell spacing and cell wall width of the square honeycomb core. These design variables are assumed to have no associated uncertainty. There are two noise factors pertaining to the air blast received by the panel. These factors are the peak pressure of the incoming pulse and the characteristic time of the pulse. The mean and variance of these factors are shown in Table 1. The response that we are interested in modeling is the maximum deflection of the back face sheet.

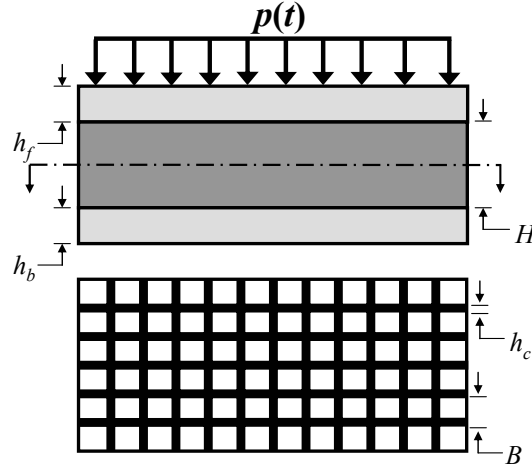


Figure 4. Schematic of BRP sandwich structure.

Table 1. Design Variables and Noise Factors

Certain Design Variables			
	Lower Bound	Upper Bound	Units
h_f	0.001	0.025	m
h_b	0.001	0.025	m
H	0.001	0.020	m
h_c	0.0001	0.010	m
B	0.005	0.050	m
Uncertain Design Variables			
	Lower Bound	Upper Bound	Units
ρ_f	2000	10000	kg/m ³
ρ_b	2000	10000	kg/m ³
ρ_c	2000	10000	kg/m ⁵
$\sigma_{Y,f}$	100	1100	MPa
$\sigma_{Y,b}$	100	1100	MPa
$\sigma_{Y,c}$	100	1100	MPa
Noise Factors			
	Mean	Variance	Units
p_0	25	3.75	MPa
t_0	0.0001	0.000015	seconds

1. Panel Deflection

Equations for the deflection of the BRP are developed based on the analysis by Xue and Hutchinson¹¹. Their analysis considered sandwich plates with square honeycomb cores and blast loads in both water and air. For the purpose of this example, only impulses in air are considered. Following the work of Taylor, the impulse load is considered to be exponential in form with a time dependence where p_0 is the peak pressure and t_0 is the pulse time¹². The impulse load acts perpendicular to the surface of the BRP and is uniformly distributed over the area of the plate. For deflection calculations, the plate is assumed to be fully clamped at both ends, of width $L/2$, and of infinite extent in the y -direction¹¹.

We extend the equations for deflection of the back face sheet developed by Xue and Hutchinson to allow for independent layer heights, and independent materials in each layer. Following the three stage deformation theory, the impulse of the blast is received by the front face sheet and momentum is transferred in stage one¹. We show the

equation for kinetic energy per unit area at the end of stage one in Equation 1. In stage two, some of the kinetic energy is dissipated through crushing of the core layer. We show the equation for the amount of kinetic energy per unit area at the end of stage two in Equation 2. The crushing strain is used to determine the crushed height of the core layer and is derived by equating the plastic dissipation per unit area in the core to the loss of kinetic energy per unit area in stage two¹¹. We show the crushing strain in Equation 3.

$$KE_I = \frac{2\rho_0^2 t_0^2}{(\rho_f \sigma_{Y,f})} \quad (\text{Eq. 1})$$

$$KE_{II} = \frac{2\rho_0^2 t_0^2}{(\rho_f \sigma_{Y,f} + \rho_c R_c \sigma_{Y,c} + \rho_b \sigma_{Y,b})} \quad (\text{Eq. 2})$$

$$\bar{\epsilon}_c = \frac{2\rho_0^2 t_0^2 (\rho_b \sigma_{Y,b} + \rho_c R_c \sigma_{Y,c})}{\lambda_c R_c \sigma_{Y,c} H \rho_f \sigma_{Y,f} (\rho_f \sigma_{Y,f} + \rho_b \sigma_{Y,b} + \rho_c R_c \sigma_{Y,c})} \quad (\text{Eq. 3})$$

In stage three, the remaining kinetic energy must be dissipated through bending and stretching of the back face sheet. The equation for deflection is derived by equating the remaining kinetic energy per unit area to the plastic work per unit area dissipated through bending and stretching. The average plastic work per unit area dissipated in stage three is estimated by summing the dissipation from bending and stretching, following the work of Xue and Hutchinson¹¹. The equation for this estimate is shown in Equation 4. The equation for deflection is shown in Equation 5.

$$W_{III}^P = \frac{2}{3} [\sigma_{Y,f} h_f + \sigma_{Y,c} R_c H \lambda_s + \sigma_{Y,b} h_b] \left(\frac{\delta}{L} \right)^2 + 4\sigma_{Y,b} h_b \frac{\bar{H}}{L} \left(\frac{\delta}{L} \right) \quad (\text{Eq. 4})$$

$$\delta = \frac{-3(\sigma_{Y,b} h_b \bar{H}) \pm \sqrt{9\sigma_{Y,b}^2 h_b^2 \bar{H}^2 \left(\frac{1}{L^2} \right) + \left(\frac{3I_0^2 [\sigma_{Y,f} h_f + \sigma_{Y,c} R_c H \lambda_s + \sigma_{Y,b} h_b]}{(\rho_f h_f + \rho_c R_c H + \rho_b h_b)} \right)}}{([\sigma_{Y,f} h_f + \sigma_{Y,c} R_c H \lambda_s + \sigma_{Y,b} h_b])} \quad (\text{Eq. 5})$$

$$\text{where } \bar{H} = H(1 - \bar{\epsilon}_c) = \left(H - \frac{2I_0^2 (\rho_b h_b + \rho_c R_c H)}{\lambda_c R_c \sigma_{Y,c} \rho_f h_f (\rho_f h_f + \rho_b h_b + \rho_c R_c H)} \right)$$

Equations for the variation in deflection are also needed in order to determine the sensitivity of the panel to variation in noise factors and uncertain design variables. The derived equation for the variation in deflection is shown in Equation 6.

$$\Delta\delta = \left| \frac{\partial\delta}{\partial\sigma_{Y,b}} \right| \Delta\sigma_{Y,b} + \left| \frac{\partial\delta}{\partial\sigma_{Y,c}} \right| \Delta\sigma_{Y,c} + \left| \frac{\partial\delta}{\partial\sigma_{Y,f}} \right| \Delta\sigma_{Y,f} + \left| \frac{\partial\delta}{\partial\rho_b} \right| \Delta\rho_b + \left| \frac{\partial\delta}{\partial\rho_c} \right| \Delta\rho_c + \left| \frac{\partial\delta}{\partial\rho_f} \right| \Delta\rho_f + \left| \frac{\partial\delta}{\partial p_0} \right| \Delta p_0 + \left| \frac{\partial\delta}{\partial t_0} \right| \Delta t_0 \quad (\text{Eq. 6})$$

2. System Constraints

We constrain the BRP design to limit mass and deflection, prohibit failure, and maintain the bounds of the design space. All the constraints that are a function of uncertain control or noise factors are imposed as "robust constraints". That is, the variation of these quantities as a result of variation in the uncertain factors is included

when evaluating the constraints. We now proceed to explain the system constraints and derive the equations for evaluation of these constraints.

We impose a maximum mass per unit area constraint to limit the panel to 100 kg/m^2 . In addition, we define the maximum allowable deflection as 10% of the span. Although the goal of the design is to minimize deflection, this constraint is imposed to keep the deflection small enough so that assumptions in the response model will not be violated. The derivation of the maximum mass per unit area constraint is shown in Equation 7, and the derivation of the maximum deflection constraint is shown in Equation 8.

$$\begin{aligned}
M - 100 \text{ kg/m}^2 &\leq 0 \\
M &= f(\rho_f, \rho_c, \rho_b, B, H, h_c, h_f, h_b) \\
g_M &= f(\rho_f, \rho_c, \rho_b, B, H, h_c, h_f, h_b) - 100 \text{ kg/m}^2 \\
\Delta g_M &= \left| \frac{\partial g_M}{\partial \rho_f} \right| \cdot \Delta \rho_f + \left| \frac{\partial g_M}{\partial \rho_b} \right| \cdot \Delta \rho_b + \left| \frac{\partial g_M}{\partial \rho_c} \right| \cdot \Delta \rho_c \\
g_M + \Delta g_M &\leq 0
\end{aligned} \tag{Eq. 7}$$

$$\begin{aligned}
\delta - 0.1L &\leq 0 \\
\delta &= f(p_0, t_0, \sigma_{y,b}, \rho_b, \sigma_{y,c}, \rho_c, \sigma_{y,f}, \rho_f, B, H, h_c, h_f, L) \\
g_\delta &= f(p_0, t_0, \sigma_{y,b}, \rho_b, \sigma_{y,c}, \rho_c, \sigma_{y,f}, \rho_f, B, H, h_c, h_f, L) - (0.1)L \\
\Delta g_\delta &= \left| \frac{\partial g_\delta}{\partial p_0} \right| \cdot \Delta p_0 + \left| \frac{\partial g_\delta}{\partial t_0} \right| \cdot \Delta t_0 + \left| \frac{\partial g_\delta}{\partial \sigma_{y,b}} \right| \cdot \Delta \sigma_{y,b} + \left| \frac{\partial g_\delta}{\partial \rho_b} \right| \cdot \Delta \rho_b + \\
&\quad \left| \frac{\partial g_\delta}{\partial \sigma_{y,c}} \right| \cdot \Delta \sigma_{y,c} + \left| \frac{\partial g_\delta}{\partial \rho_c} \right| \cdot \Delta \rho_c + \left| \frac{\partial g_\delta}{\partial \sigma_{y,f}} \right| \cdot \Delta \sigma_{y,f} + \left| \frac{\partial g_\delta}{\partial \rho_f} \right| \cdot \Delta \rho_f \\
g_\delta + \Delta g_\delta &\leq 0
\end{aligned} \tag{Eq. 8}$$

We impose a minimum relative density of the core to ensure crushing of the core rather than buckling, which would not dissipate as much energy. The relative density is not a function of uncertain factors. The equation for the calculation of the relative density constraint is shown in Equation 9.

$$\begin{aligned}
R_c &= (2Bh_c - h_c^2) / B^2 \\
0.07 - R_c &\leq 0
\end{aligned} \tag{Eq. 9}$$

We impose two constraints to avoid shear failure of the front face sheet. The first criterion prohibits shear of the face sheet at the clamped ends of the plate, and the second criterion prohibits shear of the face sheet at the core webs. These constraints are shown in Equations 10 and 11, respectively.

$$\begin{aligned}
g_{SH1} &= (2p_0 t_0 / h_f \sqrt{\sigma_{y,f} \rho_f}) - \Gamma_{SH}, \text{ where } \Gamma_{SH} = 0.6 \\
\Delta g_{SH1} &= \left| \frac{\partial g_{SH1}}{\partial p_0} \right| \Delta p_0 + \left| \frac{\partial g_{SH1}}{\partial t_0} \right| \Delta t_0 + \left| \frac{\partial g_{SH1}}{\partial \sigma_{y,f}} \right| \Delta \sigma_{y,f} + \left| \frac{\partial g_{SH1}}{\partial \rho_f} \right| \Delta \rho_f \\
g_{SH1} + \Delta g_{SH1} &\leq 0
\end{aligned} \tag{Eq. 10}$$

$$\begin{aligned}
g_{SH2} &= \frac{\rho_c HR_c}{\rho_f h_f} - \frac{4}{\sqrt{3}} \\
\Delta g_{SH2} &= \left| \frac{\partial g_{SH2}}{\partial \rho_f} \right| \Delta \rho_f + \left| \frac{\partial g_{SH2}}{\partial \rho_c} \right| \Delta \rho_c \\
g_{SH2} + \Delta g_{SH2} &\leq 0
\end{aligned} \tag{Eq. 11}$$

Finally, we impose constraints to keep the material property design variables within the specified bounds in spite of the assumed uncertainty in these variables. We show a generic form of these constraints in Equation 12. There are two such constraints for each of the six uncertain design variables.

$$\begin{aligned}
(\text{lower bound}) - \left(x - \frac{1}{2} \Delta x\right) &\leq 0 \\
\left(x + \frac{1}{2} \Delta x\right) - (\text{upper bound}) &\leq 0 \\
\forall x &= \{\rho_b, \rho_c, \rho_f, \sigma_{Y,b}, \sigma_{Y,c}, \sigma_{Y,f}\}
\end{aligned} \tag{Eq. 12}$$

B. Formulation of the Deviation Function

With the governing equations and system constraints identified, we now formulate equations for the system goals. In robust design there are at least two goals: bring the mean on target, and minimize variance. In this example, the target for deflection of the BRP is as small as possible, i.e., the deflection of the panel should be minimized. Equations for the goals are formulated by normalizing the achievement of those goals by a target value and adding deviation variables for over- and under-achievement of the goals.

In Equations 13 and 14, we show the goals for the BRP design. Both cases reflect a minimizing goal, and as such, the targets are divided by the achievement. For a maximizing goal, the achievement is divided by the target. The deviation of the normalized value from unity is represented by the deviation variables⁶.

$$T_\delta / \delta + d_1^- - d_1^+ = 1 \tag{Eq. 13}$$

$$T_{\Delta\delta} / \Delta\delta + d_2^- - d_2^+ = 1 \tag{Eq. 14}$$

The deviation variables must all be greater than or equal to zero, and the product of the two deviation variables for a goal must be equal to zero⁶. Because of these constraints, one of the two deviation variables will always be equal to zero. Since the goals are minimizing goals, the over-achievement (d_i^+) will always be zero, and the under-achievement (d_i^-) is used in the deviation function.

Due to its ease of use, the Archimedian formulation of the deviation function is chosen. It consists of a weighted sum of the deviation variables, and the weights are chosen such that they are all greater than or equal to zero and sum to unity. These weights allow for a cDSP to be run for several different scenarios. The deviation function for the BRP design is shown in Equation 15. We seek to minimize the value of Z in solving the cDSP in order to find the design point with the smallest deviation from the goals.

$$Z = W_1 \cdot d_1^- + W_2 \cdot d_2^- \tag{Eq. 15}$$

C. Scenarios for the BRP Design

To analyze the robustness of the BRP design to the uncertain parameters, three design scenarios are identified. The *robust* scenario places equal priority on the goals of minimizing deflection and minimizing variance. The *optimizing* scenario places all weight on the minimizing deflection goal and zero weight on the minimizing variance goal. Finally, the *stabilizing* scenario places all weight on the minimizing variance goal and zero weight on the minimizing deflection goal. In addition, to analyze the effect of the mass constraint, we seek to solve the cDSP at three values of the mass per unit area constraint. With the design scenarios thus defined, we present the math formulation of the BRP cDSP in Figure 5.

Given:		
A impulse load defined by peak pressure, p_0 , and characteristic time, t_0		
Mean and variance of noise factors (t_0, p_0)		$\mu_t = 10^{-4}$ seconds, $\Delta t = 0.15 \cdot \mu_t$ $\mu_p = 25$ MPa, $\Delta p = 0.15 \cdot \mu_p$
Variance of uncertain design variables (ρ, σ_Y)		$\Delta \rho = (0.05)[\text{upper bound} - \text{lower bound}]$ $\Delta \sigma_Y = (0.05)[\text{upper bound} - \text{lower bound}]$
Target for deflection (smaller-the-better)		$T_\delta = 0.05$ m
Target for variance of deflection (smaller-the-better)		$T_{\Delta\delta} = 0.005$ m
Area = 1 m^2		
Core crushing and stretching factors for a square honeycomb		$\lambda_c = 1, \lambda_s = 0.5,$
BRP Deflection Model	$[\delta, \Delta\delta, M, \Delta M] = f(\sigma_{Y,b}, \sigma_{Y,c}, \sigma_{Y,f}, \rho_b, \rho_c, \rho_f, p_0, t_0, B, H, h_c, h_f, h_b, L)$	
Find:		
Material properties:		$\sigma_{Y,f}, \rho_f, \sigma_{Y,b}, \rho_b, \sigma_{Y,c}, \rho_c$
Core geometry:		h_c, B
Height of each layer:		h_f, h_b, H
Value of deviation variables		$d_i^+, d_i^- (i = 1,2)$
Satisfy:		
<i>Constraints</i>		
Mass/area of BRP must not exceed 100 kg/m^2		$g_M + \Delta g_M \leq 0$
Deflection must not exceed 10% of span for specified boundary conditions		$g_\delta + \Delta g_\delta \leq 0$
Relative Density must be greater than 0.07 to avoid buckling		$R_c - 0.07 \leq 0$
Front face shear-off constraints		$g_{SH1} + \Delta g_{SH1} \leq 0$ $g_{SH2} + \Delta g_{SH2} \leq 0$
Uncertain control variables must stay within bounds		$(\text{lower bound}) - (x - \frac{1}{2}\Delta x) \leq 0$ $(x - \frac{1}{2}\Delta x) - (\text{upper bound}) \leq 0$ $\forall x = \{\rho_b, \rho_c, \rho_f, \sigma_{Y,b}, \sigma_{Y,c}, \sigma_{Y,f}\}$
Deviation variables must be greater than or equal to zero and multiply to zero		$d_i^+ \cdot d_i^- = 0, \text{ with } d_i^+, d_i^- \geq 0 (i = 1,2)$
<i>Goals</i>		
Minimize deflection (smaller-the-better)		$T_\delta / \delta + d_1^- - d_1^+ = 1$
Minimize variance of deflection (smaller-the-better)		$T_{\Delta\delta} / \Delta\delta + d_2^- - d_2^+ = 1$
<i>Bounds</i>		
$(0.1 \text{ mm} \leq h_c \leq 1 \text{ cm})$	$(100 \text{ MPa} \leq \sigma_{Y,b} \leq 1100 \text{ MPa})$	$(2000 \text{ kg/m}^3 \leq \rho_b \leq 10,000 \text{ kg/m}^3)$
$(5 \text{ mm} \leq H \leq 5 \text{ cm})$	$(100 \text{ MPa} \leq \sigma_{Y,f} \leq 1100 \text{ MPa})$	$(2000 \text{ kg/m}^3 \leq \rho_c \leq 10,000 \text{ kg/m}^3)$
$(1 \text{ mm} \leq B \leq 2 \text{ cm})$	$(100 \text{ MPa} \leq \sigma_{Y,c} \leq 1100 \text{ MPa})$	$(2000 \text{ kg/m}^3 \leq \rho_f \leq 10,000 \text{ kg/m}^3)$
$(1 \text{ mm} \leq h_f \leq 2.5 \text{ cm})$		
$(1 \text{ mm} \leq h_b \leq 2.5 \text{ cm})$		
Minimize:		
Deviation Function:		$Z = W_1 \cdot d_1^- + W_2 \cdot d_2^+$
Scenario 1: Minimize deflection only		$W_1 = 1, W_2 = 0$
Scenario 2: Minimize variance of deflection only		$W_1 = 0, W_2 = 1$
Scenario 3: Equal priority		$W_1 = 0.5, W_2 = 0.5$

Figure 5. Math Formulation of BRP cDSP

D. Results and Discussion

1. Verifying the cDSP Results

Solutions for the three design scenarios were found by implementing the cDSP shown above using the `fmincon` function in MATLAB¹³. This function uses a gradient based method that may return only local solutions¹⁴. To analyze the solutions found using this function, we perform an initial value analysis. The initial values for the optimization function were defined at three points in the range of the upper and lower bounds of the design variables. Solutions were found at the quarter point, half point and three quarter point of the range. If the same constraints are active for each of the three initial values, and if the routine converges to similar values of the objective function, we are confident that the results of the cDSP are valid. We show the table of active constraints in Table 2, and the convergence to a solution in Figure 6. The constraints that are active for each solution are denoted by a boldface value in the starting point columns.

Table 2. Active Constraints

	Units	Constraint Type	Bounds		Starting Point		
			Lower	Upper	Lower	Mid	Upper
BRP Dimensions							
Cell Spacing, B	mm	between	1	20	20	20	20
Core Height, H	mm	between	5	50	15.767	14.4633	13.0352
Cell Wall Thickness, h _c	mm	between	0.1	10	2.2925	3.9364	6.2386
Front Face Sheet Thickness, h _f	mm	between	1	25	13.443	8.3285	6.5977
Back Face Sheet Thickness, h _b	mm	between	1	25	21.6111	25	25
Worst Case Material Properties							
Yield Strength, Back	MPa	between	100	1100	1100	1100	1100
Yield Strength, Core	MPa	between	100	1100	1100	1100	1100
Yield Strength, Front	MPa	between	100	1100	1100	1100	1100
Density, Back	kg/m ³	between	2000	10000	2000	2000	2000
Density, Core	kg/m ³	between	2000	10000	2000	2000	2000
Density, Front	kg/m ³	between	2000	10000	2000	2000	2000
Worst Case Constraint Analysis							
Mass	kg/m ²	at most		100	100	100	100
Deflection	m	at most		0.100	0.0905	0.0912	0.0929
Relative Density of the Core	no unit	at least	0.070		0.2161	0.3549	0.5266
Mu	no unit	at most		2.309	0.2535	0.6163	1.0403
Gamma	no unit	at most		0.600	0.1969	0.3178	0.4012

The active constraints are the same for each initial value, with the exception of the back face sheet thickness for the lower starting point. The solutions are bounded by the upper bound of the cell spacing, the upper bound of the back face sheet thickness, the upper bound of the yield strength for each layer, the lower bound of the density for each layer and the maximum allowable mass per area. These nine active constraints almost fully constrain this design since there are eleven design variables. In the convergence plot, we see that all three solutions smoothly converge to a solution, and the solutions are all very similar in value. This evidence, along with the verification of the active constraints, builds our confidence that we are achieving appropriate solutions by running the cDSP. We now turn to an analysis of the solutions for the three design scenarios.

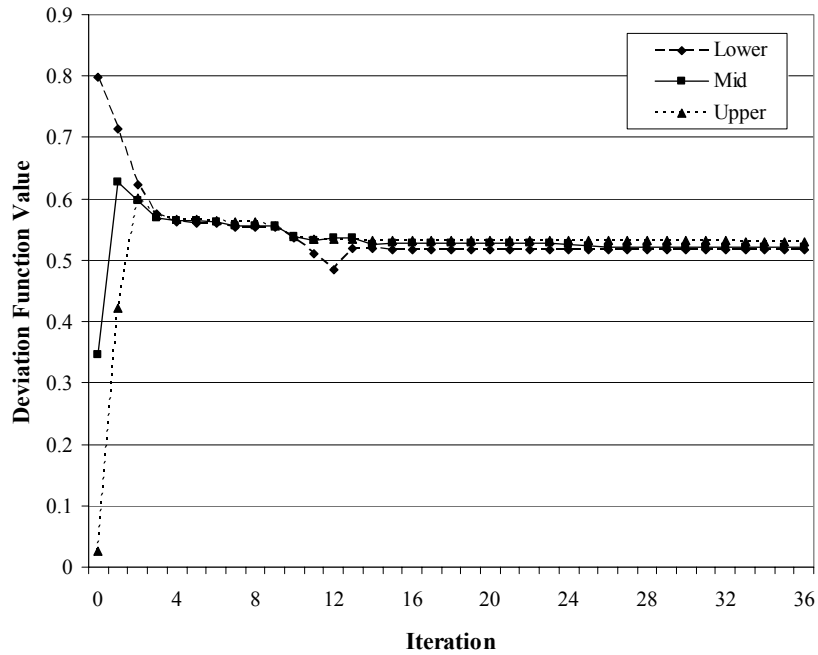


Figure 6. Convergence to Solution

2. *Comparing the Scenarios: What is a Robust Solution?*

The values of the design variables for each scenario are shown in Table 3 and Table 4, and predicted values for the achievement of the two goals are shown in Table 5.

Table 3. Values of Material Property Design Variables for Three Scenarios

	Yield Strength (MPa)			Density (kg/m ³)		
	Back	Core	Front	Back	Core	Front
Scenario 1: Optimizing	1075.00	1075.00	1075.00	2200.00	2200.00	2200.00
Scenario 2: Stabilizing	1075.00	1062.02	823.53	2434.23	3142.39	2200.00
Scenario 3: Robust	1075.00	1075.00	1075.00	2200.00	2200.00	2200.00

Table 4. Values of Geometric Design Variables for Three Scenarios

	Layer Height (mm)			Core Topology	
	Back	Core	Front	B (mm)	h_c (mm)
Scenario 1: Optimizing	25.000	10.506	9.606	20.000	4.088
Scenario 2: Stabilizing	24.989	33.882	5.357	11.659	0.744
Scenario 3: Robust	25.000	14.463	8.329	20.000	3.936

Table 5. Predicted Deflection and Variance for Three Scenarios

	Deflection (m)	Variance of Deflection (m)	Total (m)
Scenario 1: Optimizing	0.0652	0.0279	0.0931
Scenario 2: Stabilizing	0.0879	0.0121	0.1000
Scenario 3: Robust	0.0661	0.0251	0.0912

In Figure 7, we show a comparison of the predicted deflection and variance of deflection of the design solution for each scenario. The variance of deflection is shown at the top of the bar to indicate the amount of variance possible in the deflection of the panel. The total height of the bars reflects the worst-case deflection. Dashed lines

are added to compare the values for the three scenarios. The dashed lines are even with the levels for the first design scenario. As expected, the solution in Scenario 1 has the smallest deflection and the largest variance because all the weight in that scenario is placed on the minimizing deflection goal. In Scenario 2, the solution has a smaller predicted variance, but the predicted deflection is so large that the predicted worst-case deflection reaches the design constraint of 0.10 meters. This is expected because all the weight in the scenario is placed on the minimizing variance goal, and there is no weight on the minimizing deflection goal. Scenario 3 represents a compromise between the two goals because equal weight was placed on both goals. This results in a smaller deflection than the stabilizing scenario and a smaller variance than the optimizing scenario, as shown in the figure. This solution is robust because it is impervious to noise (minimizes variance of deflection) while also meeting the performance target (minimizes deflection). The stabilizing scenario solution is also impervious to noise, but does not meet the performance goal of minimizing deflection. The optimizing scenario solution meets the performance goal, but is more sensitive to uncertainty. The robust solution is preferred because it balances the two goals of minimizing deflection and minimizing variance. As a result, the solution is less sensitive to the uncertain factors, and the predicted deflection of the robust solution is only very slightly larger than the predicted deflection of the optimizing solution. And as a result of the smaller variance, the worst-case deflection of the robust solution is less than the worst-case deflection of the optimizing solution.

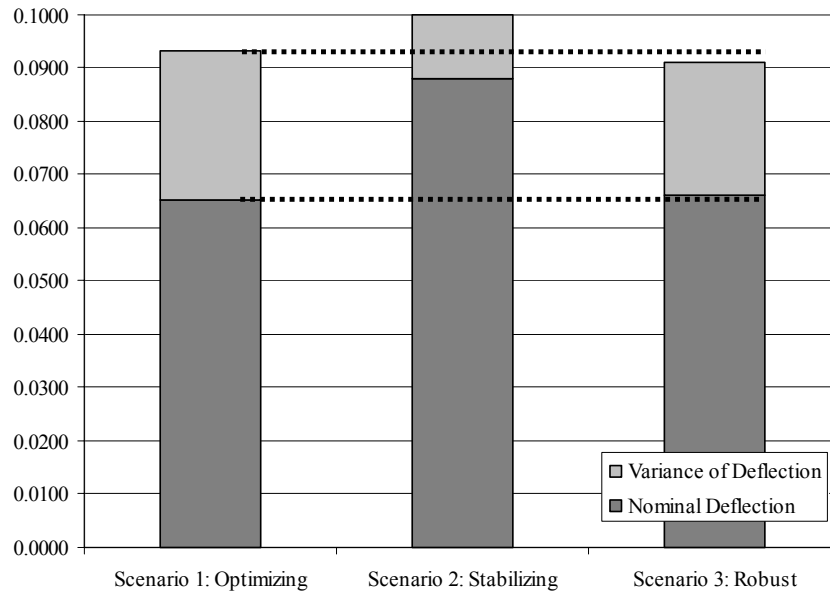


Figure 7. A Comparison of Optimizing, Stabilizing, and Robust Design Scenarios

3. Investigating the Need for New Materials for BRPs

As is shown in Table 3, the yield strength design variables tend toward the high end of the bounds and the density design variables tend toward the low end of the bounds in the robust scenario. This is not surprising because a stronger (higher yield strength) material would resist deformation to a greater extent than a weaker (lower yield strength) material. Also, a lighter (lower density) material allows for more material than a heavier (higher density) material without violating the mass constraint. This brings us back to our motivation: is there a need for a designed material for the BRP?

There are two cases to consider in the answer to this question. The first case is one in which a material is selected or designed separately for each layer of the BRP. For this case we look at the ranges of material properties in each layer individually and compare those ranges to the properties of existing materials. The second case is one in which a single material is used for all three layers of the BRP. For this case we must first find the ranges of material properties that are common to the three layers. Then we compare the properties of existing materials to this common range. The ranges of material properties for each layer and the common ranges of material properties are shown in Table 6. The common ranges are found by taking the minimum value of the three upper bounds and the maximum value of the three lower bounds. In this example, since the material property values for all three layers are the same, the common range is the same as the ranges for each of the layers. A search of a material property

database reveals no matches that fit any of these material property ranges¹⁵; hence, the materials for the BRP must be designed, or the problem formulation must be reevaluated.

Table 6. Ranges of Material Properties for Each Layer and in Common

	Yield Strength (MPa)		Density (kg/m ³)	
	Lower	Upper	Lower	Upper
Back	1050	1100	2000	2400
Core	1050	1100	2000	2400
Front	1050	1100	2000	2400
Common	1050	1100	2000	2400

4. Investigating the Impact of the Mass Constraint

To investigate the role of the mass-per-area constraint, robust solutions were found for two additional values of maximum mass-per-area: 85 kg/m² and 115 kg/m². The values of the design variables are shown in Table 7 and Table 8, and predicted values for the achievement of the two goals are shown in Table 9. As shown in Table 7, the values of the material property design variables are not significantly affected by increasing the mass-per-area constraint to 115 kg/m²; however, the material property design variables do change when the mass per area constraint is reduced to 85 kg/m². When the mass per area constraint is reduced, the yield strength needed in the core is much less than the strength of the front and back face sheets. In addition, the density of the core is larger than that of the face sheets. In Table 8 we find that the thickness of the back face sheet increases slightly with the mass constraint until it reaches its upper bound. The height of the core layer decreases with the increase in mass, while the thickness of the front face sheet increases. This is expected because the larger allowable mass enables thicker face sheets, which are stronger than thin face sheets. Since the face sheets are stronger, the core does not need to dissipate as much energy, and can be thinner. The thickness of the core cell walls increases with an increase in the mass constraint; however, the cell spacing also increases, resulting in fewer cells throughout the core. By looking at Table 9, we see that smaller values for deflection and variance of deflection are predicted as the value of the mass constraint increases.

Table 7. Values of Material Property Design Variables for Three Levels of the Mass-per-area Constraint

Mass per Area Constraint	Yield Strength (MPa)			Density (kg/m ³)		
	Back	Core	Front	Back	Core	Front
85	1073.29	676.56	1074.80	2226.94	2586.78	2211.02
100	1075.00	1075.00	1075.00	2200.00	2200.00	2200.00
115	1075.00	1075.00	1075.00	2200.00	2200.00	2200.00

Table 8. Values of Geometric Design Variables for Three Levels of the Mass-per-area Constraint

Mass per Area Constraint	Layer Height (mm)			Core Topology	
	Back	Core	Front	B (mm)	h_c (mm)
85	24.427	44.594	4.406	14.898	0.531
100	25.000	14.463	8.329	20.000	3.936
115	25.000	13.406	14.318	20.000	4.081

Table 9. Achievement of Design Goals for Three Levels of the Mass-per-area Constraint

Mass per Area Constraint	Deflection (m)	Variance of Deflection (m)	Total (m)
85	0.0856	0.0139	0.0995
100	0.0661	0.0251	0.0912
115	0.0570	0.0221	0.0791

IV. Closure

In this paper, we have presented a methodology for the design of a blast resistant panel that is robust to uncertainty in loading conditions and material property design variables. The cDSP was used as the mathematical construct for balancing the goals of minimizing deflection and minimizing the variance of deflection. Three

scenarios were presented to show the differences between robust and optimum designs, and robust ranges of acceptable material properties were identified for the robust solution.

By executing this example design problem, we have shown that the cDSP can be used to determine ranges of material properties that meet performance goals by assuming the properties of the material to be independent design variables with associated uncertainty. These ranges can be used to investigate the need for material design in the context of a product design. In this paper, the level of uncertainty in material property design variables was chosen to be 5% of the range of those design variables; however, the amount of uncertainty in those variables will have an impact on the quality of design solutions. In the future, the development of methods for identifying an appropriate level of uncertainty for the material property design variables, as well as metrics for determining the value of the design freedom afforded by these uncertain design parameters is warranted.

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References

- ¹Fleck, N. A. and Deshpande, V. S., "The Resistance of Clamped Sandwich Beams to Shock Loading," *Journal of Applied Mechanics*, 2004. Vol. 71: p. 386-401.
- ²Taguchi, G., *Introduction to Quality Engineering*. Asian Productivity Organization, Tokyo, 1986.
- ³Taguchi, G. and Clausing, D., "Robust Quality," *Harvard Business Review*, 1990. Vol. Jan/Feb: p. 65-75.
- ⁴Seepersad, C. C., "A Robust Topological Preliminary Design Exploration Method with Materials Design Applications," Ph.D. Dissertation, G. W. Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA, 2004.
- ⁵Mistree, F., Smith, and Bras, B. A., "A Decision Based Approach to Concurrent Engineering," *Handbook of Concurrent Engineering*, edited by H.R. Paresai and W. Sullivan, Chapman & Hall, New York, 1993, pp. 127-158.
- ⁶Mistree, F., Hughes, O. F., and Bras, B. A., "The Compromise Decision Support Problem and the Adaptive Linear Programming Algorithm," *Structural Optimization: Status and Promise*, edited by M.P. Kamat, AIAA, Washington, D.C., 1993, pp. 247-286.
- ⁷Zenkert, D., *An Introduction to Sandwich Construction*. Engineering Materials Advisory Services, London, 1995.
- ⁸Zenkert, D., *The Handbook of Sandwich Construction*. Engineering Materials Advisory Services, London, 1997.
- ⁹Xue, Z. and Hutchinson, J. W., "Preliminary assessment of sandwich plates subject to blast loads," *International Journal of Mechanical Sciences*, 2003. Vol. 45: p. 678-705.
- ¹⁰Xue, Z. and Hutchinson, J. W., "A comparative study of impulse-resistant metal sandwich plates," *International Journal of Impact Engineering*, 2004. Vol. 30: p. 1283-1305.
- ¹¹Xue, Z. and Hutchinson, J. W., "Metal sandwich plates optimized for pressure impulses," *International Journal of Mechanical Sciences*, 2005. Vol. 47: p. 545-569.
- ¹²Taylor, G. I., "The pressure and impulse of submarine explosion waves on plates," *The scientific papers of G. I. Taylor*, Cambridge University Press, Cambridge, 1963, pp. 287-303.
- ¹³MATLAB, Software Package, Ver. 7.0.1.24704(R14) Service Pack 1. The Mathworks, Inc, Natick, MA, 2004.
- ¹⁴The Mathworks, Inc., "fmincon," *Optimization Toolbox User's Guide* [online user's guide], Version 3, URL: www.mathworks.com/access/helpdesk/help/toolbox/optim/ug/fmincon.html [cited August 29, 2006].
- ¹⁵Automation Creations, Inc., MatWeb Material Property Data [online database], URL: www.matweb.com [cited August 9, 2006].