Robust Materials Design of Blast Resistant Panels

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Outline

- **Motivation: Materials Design vs Material Selection**
- **Solution Finding Method: cDSP**
- **Example Problem: BRP design**
  - **Robust Design**
  - **BRP Deflection Modeling**
  - **Problem Formulation**
  - **Design Scenarios**
  - **Results and Analysis**
- **Closing**

Motivation

Material Selection

- Designs are limited by the finite set of available materials

Material Design

- Tailoring the properties of materials to meet product performance goals
- Complex multiscale material models are needed, but they increase design complexity
- **Designers need a method for choosing between material design and material selection**

Robust Design

- Improving product quality by reducing sensitivity to uncertain factors
  - Noise factors
  - Uncertain control factors
  - Model uncertainty
- **Robust design techniques can be used to find design solutions that reduce sensitivity to uncertainty in material properties**, thus leaving design freedom for material design or selection

Our goal in this work:

- **Use robust design techniques to design a blast resistant panel that is robust to uncertainty in material properties and loading conditions**

The Compromise Decision Support Problem

- A general framework for solving multi-objective non-linear, optimization problems
  - Given
  - Find
  - Satisfy
  - Minimize
- Hybrid formulation based on Mathematical Programming and Goal Programming

GIVEN

An alternative to be improved through modification
Assumptions used to model the domain of interest
Tbe system parameters:

n	number of system variables
p+q	number of system constraints
p	equality constraints
q	inequality constraints
m	number of system goals
$C_i(\mathbf{X})$	Capability of the system
$D_i(\mathbf{X})$	Demand to the system
$g_i(\mathbf{X})$	System constraint function

$$g_i(\mathbf{X}) = C_i(\mathbf{X}) - D_i(\mathbf{X})$$

$f_k(d_i^+, d_i^-)$ Function of deviation variables to be minimized at priority level k the preemptive case

FIND

X_i	System Variables	i = 1,..., n
d_i^+, d_i^-	Deviation Variables	i = 1,..., m

SATISFY

System Constraints (linear, nonlinear)

$g_i(\mathbf{X}) = 0$	i = 1,..., p
$g_i(\mathbf{X}) \geq 0$	i = p+1,..., p+q

System Goals (linear, nonlinear)

$A_i(X) + d_i^- - d_i^+ = G_i$	i = 1,..., m

Bounds

$X_{i,\min} \leq X_i \leq X_{i,\max}$	i = 1,..., n
$d_i^+, d_i^- \geq 0$	i = 1,..., m
$d_i^+ \cdot d_i^- = 0$	i = 1,..., m

MINIMIZE

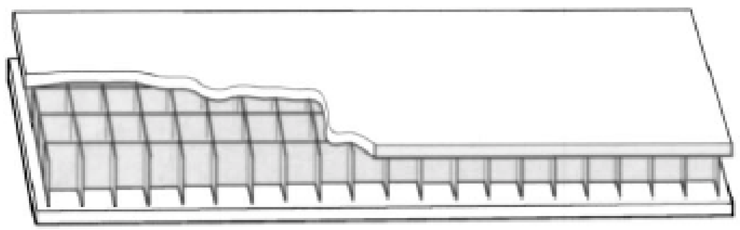
Deviation function: Archimedean formulation

$$Z = \sum_i W_i(d_i^+, d_i^-) \qquad i = 1,\ldots, m$$

Mistree, F., Smith, and Bras, B. A., "A Decision Based Approach to Concurrent Engineering," *Handbook of Concurrent Engineering*, edited by H.R. Paresai and W. Sullivan, Chapman & Hall, New York, 1993, pp. 127-158.

Mistree, F., Hughes, O. F., and Bras, B. A., "The Compromise Decision Support Problem and the Adaptive Linear Programming Algorithm," *Structural Optimization: Status and Promise*, edited by M.P. Kamat, AIAA, Washington, D.C., 1993, pp. 247-286.

Example Problem: Blast Resistant Panels (BRPs)



- **Designed to dissipate blast energy through allowable plastic deformation (crushing) of the core layer**
- **Three layer metal sandwich panel with a square honeycomb core**

- **Design objective: minimize deflection of the back face sheet while meeting mass and failure constraints**
- **Robustness : minimize variance of deflection of the back face sheet due to variation in noise factors**

Fleck, N.A. and V.S. Deshpande, 2004, "The Resistance of Clamped Sandwich Beams to Shock Loading". *Journal of Applied Mechanics,* Vol. 71, p. 386-401.

Designing a Robust Solution

Noise Factors

In order to design a BRP that is robust to the blast load, the BRP should be robust to uncertainty in t_0 and p_0.

Uncertain Control Factors

In order to maintain design freedom in the materials of the BRP, the BRP should be robust to uncertainty in yield strength and density of each layer.

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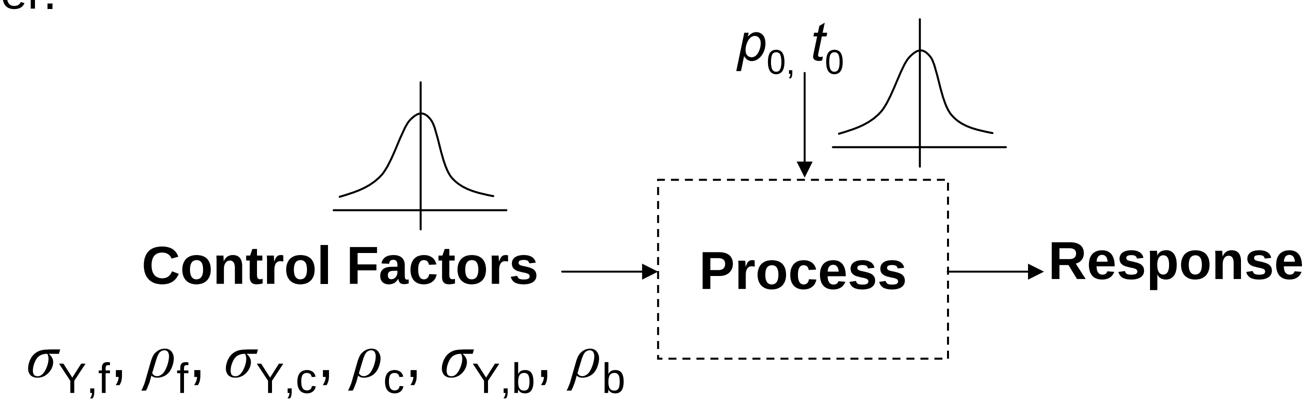
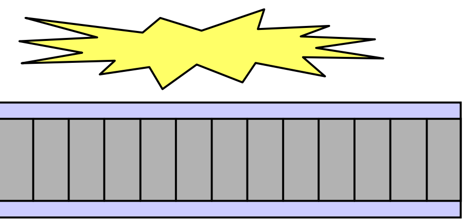


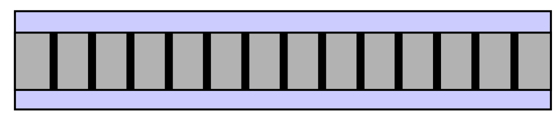
Predicting Panel Deflection:
Three-Stage Deformation

The BRP energy absorption and deformation is divided into 3 Stages:

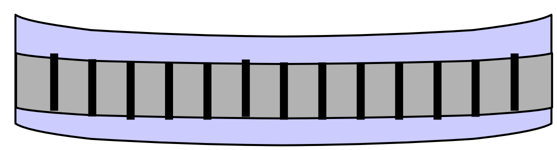
- **Stage I – Fluid-structure interaction**
  - 0 ~ 0.1ms
  - Impulse encounters top face sheet



- **Stage II – Core crushing**
  - 0.1 ~ 0.4 ms
  - Impulse energy is dissipated as the matrix of the core crushes and deforms



- **Stage III – Plastic bending and stretching**
  - 0.4 ~ 25 ms
  - All remaining impulse energy is dissipated as the back face sheet stretches and bends



Fleck, N.A. and Deshpande, V.S., 2004, "The Resistance of Clamped Sandwich Beams to Shock Loading". *Journal of Applied Mechanics*. 71: p. 386-401.

Systems
Realization
Laboratory

Predicting Panel Deflection

Stage I: Momentum of blast transferred to front face sheet

Total kinetic energy/area at end of Stage I:

$$KE_I = \frac{2p_0^2 t_0^2}{\left(\rho_f \sigma_{Y,f}\right)}$$

Stage II: Core crushing dissipates some energy

Total kinetic energy / area at end of Stage II:

$$KE_{II} = \frac{2p_0^2 t_0^2}{\left(\rho_f \sigma_{Y,f} + \rho_c R_c \sigma_{Y,c} + \rho_b \sigma_{Y,b}\right)}$$

Crushing strain:

$$\bar{\varepsilon}_c = \frac{2p_0^2 t_0^2 \left(\rho_b \sigma_{Y,b} + \rho_c R_c \sigma_{Y,c}\right)}{\lambda_c R_c \sigma_{Y,c} H \rho_f \sigma_{Y,f} \left(\rho_f \sigma_{Y,f} + \rho_b \sigma_{Y,b} + \rho_c R_c \sigma_{Y,c}\right)}$$

Xue, Z. and Hutchinson, J.W., 2005, "Metal sandwich plates optimized for pressure impulses". *International Journal of Mechanical Sciences*. 47: p. 545–569.

Nomenclature:

- ρ_f = density of front face sheet
- ρ_c = density of core
- ρ_b = density of back face sheet
- p_0 = peak pressure of impulse load
- t_0 = characteristic time of impulse load
- δ/L = deflection per length of back face sheet
- $\sigma_{y,f,}$ = yield strength of front face sheet
- $\sigma_{y,c,}$ = yield strength of core material
- $\sigma_{y,b,}$ = yield strength of back face sheet
- h_f = front face sheet thickness
- h_b = back face sheet thickness
- H = core thickness
- h_c = core cell wall thickness
- R_c = relative density of core
- λ_s = core stretching strength factor
- λ_c = core crushing strength factor

Predicting Panel Deflection

Stage III: Remaining energy is dissipated through bending and stretching of back face sheet

Plastic work/area in Stage III dissipates the kinetic energy/area remaining at the end of Stage II:

$$W_{III}^{P} = \frac{2}{3}\left[\sigma_{Y,f}h_{f} + \sigma_{Y,c}R_{c}H\lambda_{S} + \sigma_{Y,b}h_{b}\right]\left(\frac{\delta}{L}\right)^{2} + 4\sigma_{Y,b}h_{b}\frac{\overline{H}}{L}\left(\frac{\delta}{L}\right)$$

Equate plastic work/area dissipated in Stage III to the kinetic energy remaining after Stage II to solve for the deflection of the back face sheet

$$W_{III}^{P} = KE_{I} - KE_{II}$$

Xue, Z. and Hutchinson, J.W., 2005, "Metal sandwich plates optimized for pressure impulses". *International Journal of Mechanical Sciences*. 47: p. 545–569.

Predicting Panel Deflection

Deflection Equation:

$$\delta = \frac{-3\left(\sigma_{Y,b} h_b \bar{H}\right) \pm \sqrt{9\sigma_{Y,b}^2 h_b^2 \bar{H}^2 \left(\frac{1}{L^2}\right) + \left(\frac{3 I_0^2 \left[\sigma_{Y,f} h_f + \sigma_{Y,c} R_c H \lambda_S + \sigma_{Y,b} h_b\right]}{\left(\rho_f h_f + \rho_c R_c H + \rho_b h_b\right)}\right)}}{\left(\left[\sigma_{Y,f} h_f + \sigma_{Y,c} R_c H \lambda_S + \sigma_{Y,b} h_b\right]\right)}$$

$$\text{where}\quad \bar{H} = H\left(1 - \bar{\varepsilon}_c\right) = \left(H - \frac{2 I_0^2 \left(\rho_b h_b + \rho_c R_c H\right)}{\lambda_c R_c \sigma_{Y,c} \rho_f h_f \left(\rho_f h_f + \rho_b h_b + \rho_c R_c H\right)}\right)$$

Variance of Deflection:

$$\Delta\delta = \left|\frac{\partial\delta}{\partial\sigma_{Y,b}}\right|\Delta\sigma_{Y,b} + \left|\frac{\partial\delta}{\partial\sigma_{Y,c}}\right|\Delta\sigma_{Y,c} + \left|\frac{\partial\delta}{\partial\sigma_{Y,f}}\right|\Delta\sigma_{Y,f} + \left|\frac{\partial\delta}{\partial\rho_b}\right|\Delta\rho_b + \left|\frac{\partial\delta}{\partial\rho_c}\right|\Delta\rho_c + \left|\frac{\partial\delta}{\partial\rho_f}\right|\Delta\rho_f + \left|\frac{\partial\delta}{\partial p_0}\right|\Delta p_0 + \left|\frac{\partial\delta}{\partial t_0}\right|\Delta t_0$$

Compromise DSP (cDSP)

Given

A feasible alternative: 3 layer sandwich core panel with square honeycomb core

Assumptions:
- air blast impulse pressure of exponential form
- blast occurs on the order of 10^{-4} seconds
- no strain hardening

Parameters: area of panel, geometric constants for analysis, mean and variance of noise factors

Find

Design variables:
- material density
- yield strength
- layer thicknesses
- cell wall thickness
- cell spacing

Deviation variables: underachievement of goals

Satisfy

Constraints:
- mass/area less than 100 kg/m^2
- deflection less than 10% of span, relative density greater than 0.07 to avoid buckling
- constraints for front face sheet shear-off

Bounds: bounds on design variables and deviation variables

Goals:
- minimize deflection of back face sheet
- minimize variance of deflection of back face sheet

Minimize

Weighted sum of the deviation variables
- **Scenario 1**: Minimize deflection only
- **Scenario 2**: Minimize variance of deflection only
- **Scenario 3**: Equal priority on both goals

Design Variables and Bounds

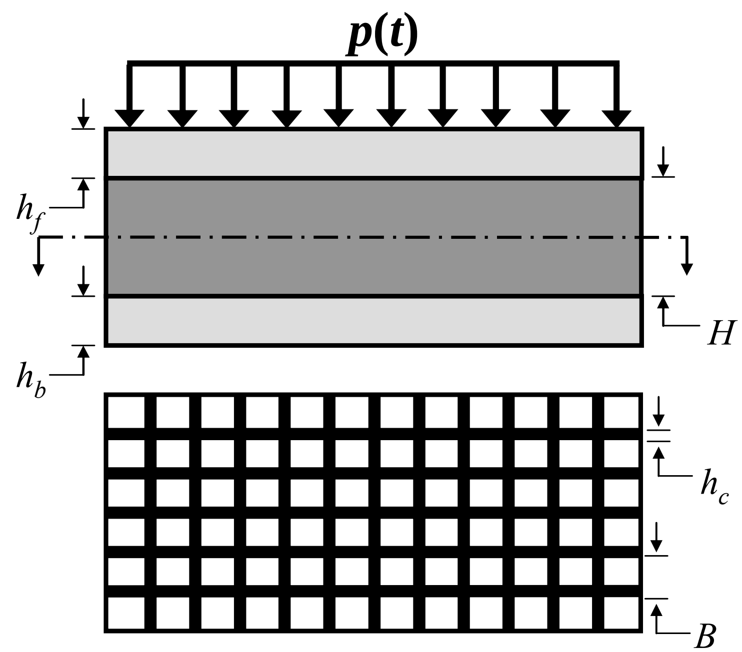
	Lower Bound	Upper Bound	Units
Certain Design Variables			
h_f	0.001	0.025	m
h_b	0.001	0.025	m
H	0.001	0.02	m
h_c	0.0001	0.01	m
B	0.005	0.05	m

Uncertain Design Variables

	Lower Bound	Upper Bound	Units
ρ_f	2000	10000	kg/m^3
ρ_b	2000	10000	kg/m^3
ρ_c	2000	10000	kg/m^3
$\sigma_{Y,f}$	100	1100	MPa
$\sigma_{Y,b}$	100	1100	MPa
$\sigma_{Y,c}$	100	1100	MPa

Noise Factors

	Mean	Variance	Units
p_0	25	3.75	MPa
t_o	0.0001	0.000015	seconds



Design Constraints

Designer / Customer Requirements:

- **Mass per Area – Less than 100 kg/m^2**
- **Back Face Sheet Deflection – Less than 10% of Length (10cm)**

Constraints to Prohibit Failure:

- **Relative Density of Honeycomb Core – Greater than 0.07 to avoid buckling**
- **Front Face Sheet Shearing – Must not fail in shear**
  - **At clamped ends**
  - **At the core webs**

Constraints on Uncertain Design Variables

- **Uncertain Material Properties – Must remain in bounds**

Robust Constraints: All constraints that are a function of uncertain factors are imposed as "robust" constraints; i.e. the worst case value including the variation due to uncertain factors is used to evaluate the constraint

Goals and Scenarios

Deviation Variables: d_i^-, d_i^+

- **Measure the deviation from the goal**
- **As we minimize the deviation variables, we approach the targets**

Goal 1: Minimize Deflection

$$T_\delta / \delta + d_1^- - d_1^+ = 1$$

$$T_\delta = 0.05 \text{ m}$$

Goal 2: Minimize Variance of Deflection

$$T_{\Delta\delta} / \Delta\delta + d_2^- - d_2^+ = 1$$

$$T_{\Delta\delta} = 0.005 \text{ m}$$

Deviation Function:

$$Z = W_1 \cdot d_1^- + W_2 \cdot d_2^-$$

Scenario 1: Optimizing

- **W1 = 1**
- **W2 = 0**

All priority on deflection goal; none on robust goal

Scenario 2: Stabilizing

- **W1 = 0**
- **W2 = 1**

All priority on robust goal; none on deflection goal

Scenario 3: Robust

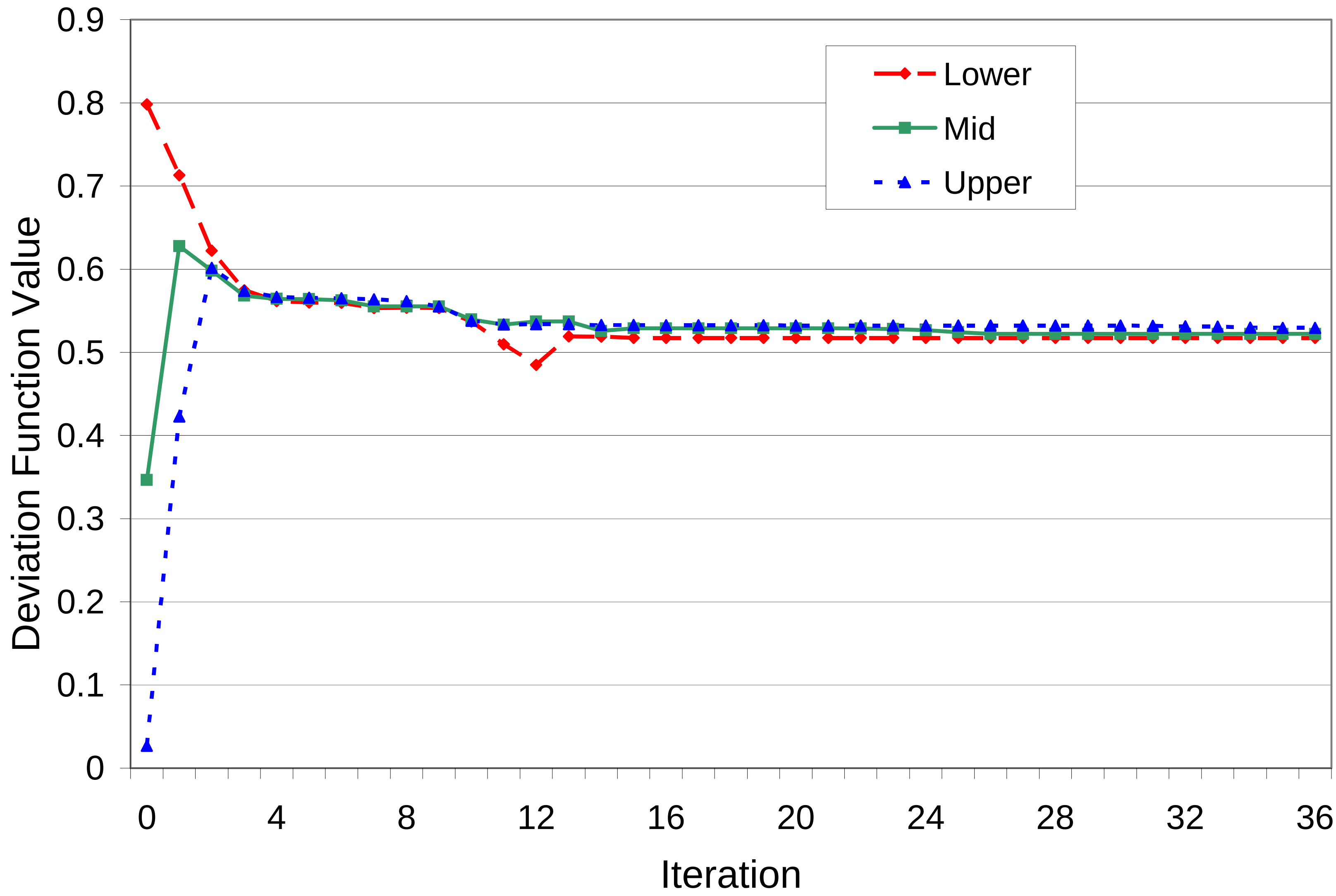
- **W1 = 0.5**
- **W2 = 0.5**

Equal priority on both goals

Verifying the cDSP Results

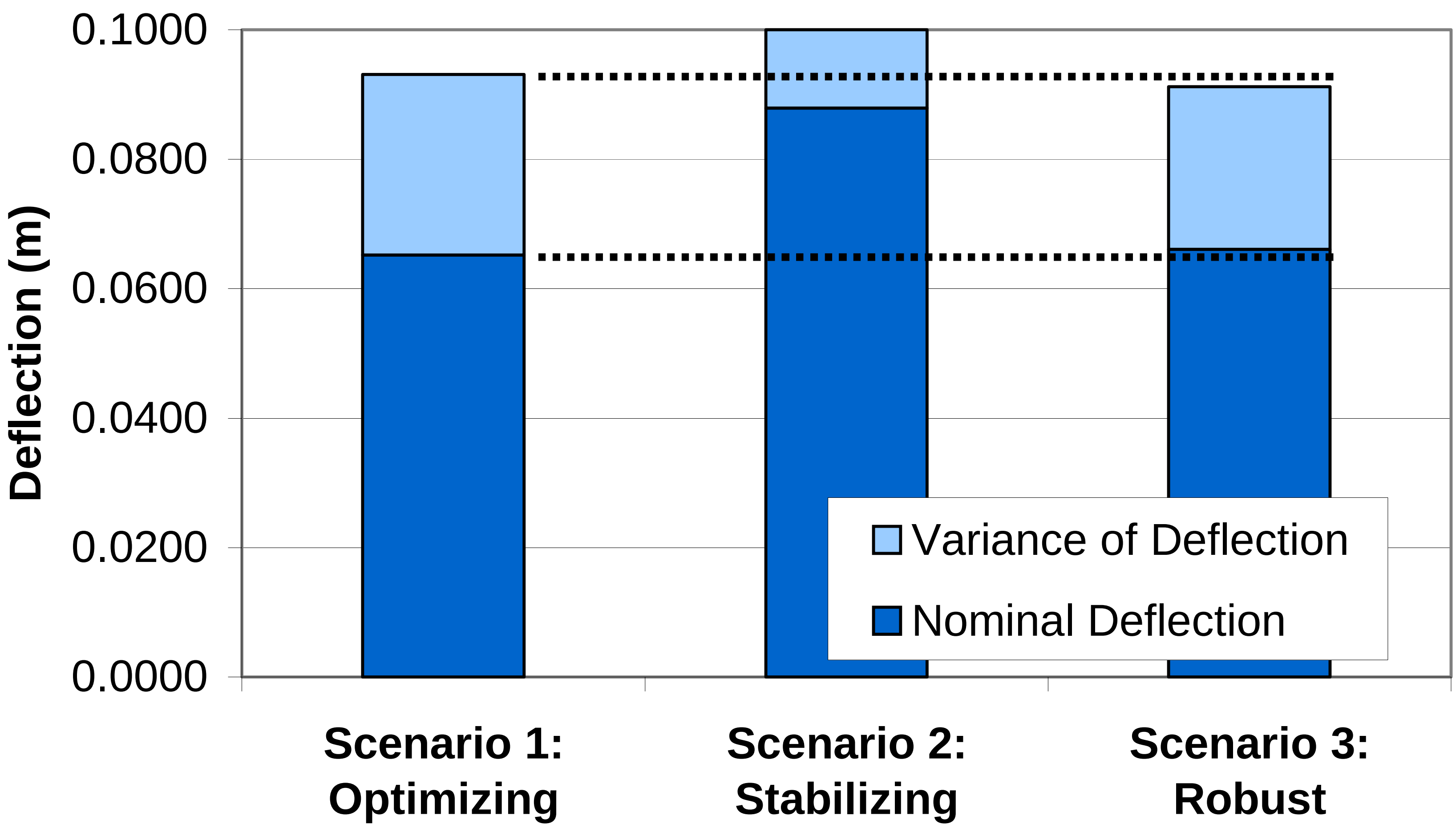
	Units	Constraint Type	Bounds Lower	Bounds Upper	Starting Point Lower	Starting Point Mid	Starting Point Upper
BRP Dimensions							
Cell Spacing, *B*	mm	between	1	20	**20**	**20**	**20**
Core Height, *H*	mm	between	5	50	15.767	14.4633	13.035
Cell Wall Thickness, h_c	mm	between	0.1	10	2.2925	3.9364	6.2386
Front Face Sheet Thickness, h_f	mm	between	1	25	13.443	8.3285	6.5977
Back Face Sheet Thickness, h_b	mm	between	1	25	21.611	**25**	**25**
Worst Case Material Properties							
Yield Strength, Back	MPa	between	100	1100	**1100**	**1100**	**1100**
Yield Strength, Core	MPa	between	100	1100	**1100**	**1100**	**1100**
Yield Strength, Front	MPa	between	100	1100	**1100**	**1100**	**1100**
Density, Back	kg/m^3	between	2000	10000	**2000**	**2000**	**2000**
Density, Core	kg/m^3	between	2000	10000	**2000**	**2000**	**2000**
Density, Front	kg/m^3	between	2000	10000	**2000**	**2000**	**2000**
Worst Case Constraint Analysis							
Mass	kg/m^2	at most		100	**100**	**100**	**100**
Deflection	m	at most		0.100	0.0905	0.0912	0.0929
Relative Density of the Core	no unit	at least	0.070		0.2161	0.3549	0.5266
Mu	no unit	at most		2.309	0.2535	0.6163	1.0403
Gamma	no unit	at most		0.600	0.1969	0.3178	0.4012

Verifying the cDSP Results



Systems Realization Laboratory

Comparing the Scenarios: What is a Robust Solution?



Investigating the Need for New Materials

All the solutions tend towards higher yield strengths and lower densities

	Yield Strength (MPa)			Density (kg/m^3)		
	Back	Core	Front	Back	Core	Front
Scenario 1: Optimizing	1075.00	1075.00	1075.00	2200.00	2200.00	2200.00
Scenario 2: Stabilizing	1075.00	1062.02	823.53	2434.23	3142.39	2200.00
Scenario 3: Robust	1075.00	1075.00	1075.00	2200.00	2200.00	2200.00

Since there are no materials that have properties that satisfy these ranges of material properties, a material must be designed or problem formulation must be reevaluated

	Yield Strength (MPa)		Density (kg/m^3)	
	Lower	Upper	Lower	Upper
Back	1050	1100	2000	2400
Core	1050	1100	2000	2400
Front	1050	1100	2000	2400
Common	1050	1100	2000	2400

Investigating the Impact of the Mass Constraint

Robust solutions were found for three values of the mass constraint

Mass per Area Constraint	Yield Strength (MPa)			Density (kg/m^3)		
	Back	Core	Front	Back	Core	Front
85	1073.29	676.56	1074.80	2226.94	2586.78	2211.02
100	1075.00	1075.00	1075.00	2200.00	2200.00	2200.00
115	1075.00	1075.00	1075.00	2200.00	2200.00	2200.00

Mass per Area Constraint	Layer Height (mm)			Core Topology (mm)	
	Back	Core	Front	*B*	h_c
85	24.427	44.594	4.406	14.898	0.531
100	25.000	14.463	8.329	20.000	3.936
115	25.000	13.406	14.318	20.000	4.081

Mass per Area Constraint	Deflection (m)	Variance of Deflection (m)	Total (m)
85	0.0856	0.0139	0.0995
100	0.0661	0.0251	0.0912
115	0.0570	0.0221	0.0791

Closure

- We have presented a methodology for the design of a blast resistant panel that is robust to uncertainty in loading conditions and material properties.

- A cDSP was formulated to balance the goals of minimizing deflection and minimizing the variance of deflection.

- Three design scenarios were presented to compare optimizing, stabilizing, and robust solutions.

- The impact of the mass constraint was examined for three values of the constraint.

Acknowledgments

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