

Parachutist Problem Solution

$$v = \frac{gm}{c} (1 - e^{-(c/m)t}) \Rightarrow \text{convert to root finding format,}$$

$$f(m) = \frac{gm}{c} (1 - e^{-(c/m)t}) - v = 0 \Rightarrow \text{plug in given values,}$$

$$f(m) = \left(\frac{9.8m}{15}\right) (1 - e^{-(15/m)9}) - 35 = 0 \Rightarrow \text{construct a table of values,}$$

i	x_{lower}	x_{upper}	$f(x_L)$	$f(x_U)$	x_r
1	50	70.000	-4.528713	4.0857326	60.61423
2	50	60.61423	-4.528713	0.288464	59.88461
3	50	59.88461	-4.528713	0.018749	59.84386

table cont...

i	$f(x_r)$	$f(x_L) \times f(x_R)$	ϵ_a
1	0.288464	-1.30637	—
2	0.018749	-0.08491	1.051%
3	0.001212	-0.00549	0.068%

\rightarrow EUREKA! It's $< 0.1\%$

$$x_r = x_u - f(x_u) \cdot \frac{(x_L - x_u)}{f(x_L) - f(x_u)}$$

$$\epsilon_a = \left| \frac{\text{current approximation} - \text{previous approximation}}{\text{current approximation}} \right| \times 100$$

check latest x_r to see if its accurate...

$$v = \frac{9.8(59.84386)}{15} (1 - e^{-(15/59.84386)9}) = 35.00121 \text{ m/s}$$

So the appropriate mass is $\boxed{m = 59.84386 \text{ kg}}$

Parachotist Newton Solution

$$f(m) = \left(\frac{9.8}{15}\right)m \left(1 - e^{-135/m}\right) - 35 = 0$$

compute derivative ...

$$u = (9.8m/15) \quad v = (1 - e^{-135/m})$$

$$du = (9.8/15) \quad dv = (-e^{-135/m})\left(\frac{135}{m^2}\right)$$

$$f'(x) = u dv + v du$$

$$f'(x) = \left(\frac{9.8m}{15}\right)\left(-e^{-135/m}\right)\left(\frac{135}{m^2}\right) + (9.8/15)\left(1 - e^{-135/m}\right)$$

$$f'(x) = \frac{(-9.8)(9)}{m} \left(e^{-135/m}\right) + (9.8/15)\left(1 - e^{-135/m}\right)$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$\epsilon_a = \left| \frac{\text{current approximation} - \text{previous approximation}}{\text{current approximation}} \right| \times 100$$

i	x_i	$f(x_i)$	$f'(x_i)$	ϵ_a
0	70	4.08573	0.37522	-
1	59.111	-0.31581	0.43473	18.42%
2	59.837	-0.0017	0.43048	1.21%
3	59.841	-1.926×10^{-5}	0.43046	0.007% \rightarrow stopping criteria met

$$\boxed{m = 59.841 \text{ kg}}$$

check by plugging in ...

$$v = \frac{(9.8)(59.841)}{15} \left(1 - e^{-135/59.841}\right) = 34.99998 \text{ m/s so } \checkmark$$