Energy-Based Modeling of a Residential Wind Turbine with Modelica

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Task 1 – Define Goals and Problem Domain

Problem Definition

This simulation study will focus on simulating energy conversion in a horizontal axis wind turbine from wind power to electrical power that can be fed directly to the utility grid. A diagram of a common wind turbine configuration is shown in Figure 1.

![Wind Turbine Diagram](image)

Figure 1: Wind Turbine Diagram [1]

There are various types of wind turbines which are characterized by different control strategies: fixed speed, variable slip, doubly-fed induction generator, and direct drive. The fixed speed turbine is chosen for this study because it is a viable option for residential usage due to reliability and robustness. A system level schematic of a fixed speed wind turbine is shown in Figure 2.
The red box in Figure 2 shows the sub-systems that will be simulated: rotor, drive train, and generator.

The rotor extracts kinetic energy from the wind and converts to rotational energy through a “low speed” shaft. The drive train links the low speed shaft to a high speed shaft via a gearbox in order to reach the required speed of the generator – 50 or 60 Hz depending on location of the turbine. An induction generator (squirrel cage induction machine) is used to convert the rotational energy of the high speed shaft into electrical energy which can be used for residential power or tie back into the utility grid. Effects of the transformer and grid are neglected in this study.

The control strategy for a fixed speed wind turbine is straight-forward, the induction generator provides necessary reaction torque to the rotor to keep a constant rotational speed in order to meet AC power transmission requirements. Also, for simplicity, the blades will be assumed as stall-regulated rather than variable pitch controlled.

The key design variables that will be investigated are:

- **Rotor radius** (m) – Increasing the rotor radius (or blade length) will increase the available wind power to be captured, i.e. the rotor will span a greater area with increased rotor radius.
- **Tower height** (m) – The height of the tower will determine the position of the rotor in the atmospheric boundary layer where wind velocity increases with distance from the ground. An increase in velocity will increase available wind power.
- **Gear ratio** (-) – Since the high speed shaft will be operating at a fixed speed (60 Hz), the turbine rotor will also be at a fixed speed. The rotational speed of the turbine rotor will be
set by the gear ratio and will have an effect on the aerodynamic performance of the blades. Optimizing the gear ratio will maximize power generation.

Each of these variables is also associated with manufacturing cost of the turbine, so an optimal configuration is desired to maximize profit of the design.

The design parameters are linked to the fundamental objective of the design through an influence diagram in Figure 3.

![Influence Diagram for Residential Scale Wind Turbine](image)

**Figure 3: Influence Diagram for Residential Scale Wind Turbine**

Performance attributes of the turbine, nominal power and annual energy rating, are calculated using the design variables. Performance and price of the turbine will ultimately determine the expected profit - the goal of this simulation is to predict performance based on various values of the design variables.
Design Scenarios

1. Steady state power output will be the primary result of interest for this simulation. The steady state power output of the turbine will be determined for wind velocity from 1 to 20 m/s (using a constant input velocity) and a power curve will be developed to verify that the behavior matches that of a fixed speed wind turbine. A typical power curve for a stall regulated turbine is shown in Figure 4.

2. Dynamic response to wind gusts will be tested in order to verify that the rotor speed is controlled and within a given tolerance. Time dependent wind velocity will be input to the rotor and the shaft speed will be monitored.

3. Effect of changing the design variables will be investigated and verified to yield expected results.

Figure 4: Power Curve for Fixed Speed Wind Turbine [3]
Task 2 – System and Simulation Specification

The model specification will be described for each sub-system (rotor, drive train, and generator) then the connection of all subsystems will be explained.

Rotor Aerodynamics

Turbine blades extract power from the wind by creating torque on the low speed shaft when air passes through the swept area of the rotor radius. A three-bladed rotor is shown in Figure 5. Wind power is defined as the kinetic energy of the wind passing through the swept area of the blades per second.

\[ P_{\text{wind}} = \frac{1}{2} \rho A V_{\text{wind}}^3 = \frac{1}{2} \rho \pi R^2 V_{\text{wind}}^3 \]  

(1)

where \( R \) is the rotor radius.

From Eq. 1, the available power increases with the square of the rotor radius and cube of the wind velocity. The wind velocity in this simulation is scaled based on the tower height using the 1/7th power law for atmospheric boundary layers:

\[ V_{\text{wind}} = V_{\text{ref}} \left( \frac{y}{y_{\text{ref}}} \right)^{\frac{1}{7}} \]  

(2)

To describe the efficiency of a turbine, a power coefficient is defined as the fraction of wind power captured in the low speed shaft:

\[ c_p = \frac{P_{\text{shaft}}}{P_{\text{wind}}} = \frac{\tau \omega_{\text{rotor}}}{P_{\text{wind}}} \]  

(3)
For an ideal wind turbine, the theoretical maximum for the power coefficient is known as the Betz limit or \( c_p = 0.5926 \). Practical wind turbines typically have a power coefficient in the range of 0.2 to 0.4.

The power coefficient can be predicted using the following equation [5]:

\[
c_p = c_1(c_2 - c_3 \beta - c_5)e^{-c_6} \quad (4)
\]

where

\[
c_1 = 0.5, c_2 = \frac{116}{\lambda}, c_3 = 0.4, c_5 = 5, c_6 = \frac{21}{\lambda}
\]

The parameters \( \lambda \) and \( \beta \) in Eq. 4 are the tip speed ratio and pitch angle of the blade respectively. The tip speed ratio is defined as

\[
\lambda = \frac{\omega_{rotor} R}{V_{wind}} \quad (5)
\]

Various curves of power coefficient vs. tip speed ratio are shown in Figure 6.

![Figure 6: Power Coefficient vs. Tip Speed ratio](image)

For a given wind velocity and gear ratio, the rotor rotational speed and tip speed ratio can be calculated:

\[
\omega_{rotor} = \frac{\omega_{gen}(Hz)}{N_{gear}} \quad (6)
\]

then use Eq. 5 to calculate tip speed ratio. The pitch of the blades was chosen to be 1° for this simulation. Equation 4 is used to predict the power coefficient and then calculate torque using
\[ \tau_{rotor} = \frac{c_p P_{\text{wind}}}{\omega_{rotor}} \quad (7) \]

Torque on the low speed shaft is the key output of the rotor aerodynamics subsystem.

**Assumptions**

- Air density is assumed to be 1.2 kg/m\(^3\). (This is a standard value for air density at sea level, altitude change will cause deviation.)
- Turbulence intensity of the wind is neglected.
  - Turbulence has a significant effect on wind turbine performance if the rotor is located near any obstacles or surrounding hills, it is assumed that the air space near the turbine is free of obstacles.
- Boundary layer velocity profile follows the \(1/7\)\(^{th}\) power law. (This is an empirical relationship commonly used to predict wind velocity at various heights above the ground.)

**Drive Train**

The drive train consists of a low speed shaft, gear box, and high speed shaft. The gear box is necessary to increase rotational speed to satisfy AC power requirements. Configuration of the drive train is shown in Figure 7.

\[ \tau_{rotor} = j\dot{\theta} + b\dot{\theta} + K\theta \quad (8) \]

The moment of inertia, \(J\), is approximated using [6]:

\[ J = 0.212 \ast (\text{Blade Mass}) \ast (\text{Blade Length})^2 \quad (9) \]
Assumptions

- Damping constant $b$ was calibrated so that 1% of the total power was dissipated in the damper at nominal conditions.

$$b = 0.07 \frac{N \cdot m \cdot s}{\text{rad}} \quad (10)$$

- Stiffness $K$ is neglected and set to 0. Data for stiffness is not readily available and it is not critical to the simulation.
- Shaft dynamics of the high speed shaft (stiffness, damping, inertia) are neglected. The rotor inertia is assumed to dominate the drive train dynamics.
- An ideal gear box is used, losses due to gears are assumed to be negligible.

Generator

Most fixed speed wind turbines use a “squirrel cage” induction generator, which is speed controlled to provide the required AC frequency. For the purposes of this study, the details of the induction generator are not simulated, a pre-packaged generator is used in Dymola which outputs a three phase signal with specified voltage (120 VAC) and frequency (60 Hz). The model works on conservation of energy:

$$\tau \omega = VI \quad (11)$$

Since the input rotational power is known and voltage is known, output current can be calculated.

Model Overview

The rotor model takes user input of wind velocity, rotor radius, and tower height and calculates torque applied to the low speed shaft. Torque is applied to the low speed shaft and stepped up to a higher speed, then linked to the generator. The generator is linked to the high speed shaft and produces 120 VAC 60 Hz power based on the given rotational power. Also, the generator must provide a resistive torque on the high speed shaft so that the rotational speed remains constant at the specified value.
Task 3 – Dymola Model

Rotor

The rotor model follows the analysis presented in task 2 to calculate torque on the low speed shaft. The Dymola model is shown in Figure 8.

![Figure 8: Rotor Dymola Model](image)

The model goes through calculation of the power coefficient, wind velocity at the tower, and finally torque.

An if statement is added to account for under and over speed, which sets the output torque to 0 if the velocity is less than 3 m/s or greater than 25 m/s.
**Drive Train**

The drive train is modeled using rotational components of inertia and damping, ideal gear with user input, and a flange connection. The drive train is shown in Figure 9.

Torque is input from the rotor model calculation and translated to rotational motion through flange b. The rotational inertia of the turbine blades and mechanical losses are modeled in the low speed shaft.

**Generator**

The generator model is based on an inverter that is included in the Modelica libraries. The Dymola model is shown in Figure 10.
The frequency and voltage (set to 120 V 60 Hz) of the electrical signal is controlled using the vf controller. This pre-packaged model allows application of torque to the induction generator and conversion of the rotational power to electrical power. A power sensor is placed to easily calculate the output power of the turbine.

*Overall Model and Interaction*

The overall model is shown in Figure 11.
The variable user inputs are shown on the right in Figure 11. Rotor radius, tower height, and wind velocity are used to calculate the torque developed by the rotor. Gear ratio is input and will determine the speed of the low speed shaft, since the speed of the high speed shaft is fixed. The generator outputs the power developed by the wind turbine.
Task 4 – Verification

Torque Calculation

The rotor model was verified by testing it over a range of wind velocity inputs. A velocity ramp was input from 0.01 m/s to 30 m/s to verify the 3 m/s cut-in and 25 m/s cut-out velocities as well as torque behavior. Figure 12 shows the test results.

![Torque Graph](image)

Figure 12: Torque (N*m) Generated for Increasing Velocity

The test results show that the torque is 0 until cut in velocity is reached then peaks and starts decreasing (due to stall controlled blades) until it hits the cut out velocity and goes to zero again.

Drive Train Test

The drive train is meant to accept a torque value and step up the shaft speed through the gear box. Torque was applied to the low speed shaft and the angular displacement of the low and high speed shaft was plotted against time with a gear ratio of 2. Figure 13 shows the results of the test.
The results indicate that the high speed shaft is rotating at twice the speed of the low speed shaft, which matches the specified gear ratio of 2.

*Generator Test*

The generator was tested by applying a constant torque to the shaft and observing the output power. A torque value of 2x greater was then applied and the output power was compared.
The electrical power output was shown to double when the torque was doubled at a constant speed. This matches expected behavior – rotational power input was doubled and the electrical power output doubled. Note, negative power indicates that the electrical power is being generated and not consumed.

However, there is strange transient behavior when the generator is started up, showing power peaking to 20 kW, this is an undesirable result and shows that further troubleshooting must be done with the generator model.
**Task 5 – Experimentation and Interpretation**

The design scenarios of task 1 (steady state power curve, response to velocity gust, and design variable testing) are investigated in this section.

*Power Curve*

Nominal design variables were chosen as

\[
R_{\text{nom}} = 3.5 \text{ m} \\
\text{Tower Height} = 30 \text{ m} \\
\text{Gear Ratio} = 8.44
\]

These values were used to define a power curve for the wind turbine to verify behavior. The power curve is shown in Figure 15.

![Power Curve for Nominal Wind Turbine Design](image)

Figure 15: Power Curve for Nominal Wind Turbine Design

The power curve shows that the wind turbine has a maximum power output of just over 9000 W near wind velocity of 15 m/s (33 mph). Also the stall regulated blade behavior shows after the peak power output. The blades are designed such that in high wind speeds, flow will separate from the airfoil and cause the rotor to become less efficient, however saving the turbine from dangerous wind loads.

Standard design rating calculations are prescribed by the American Wind Energy Association (AWEA) for small scale wind turbines [7].
- Nominal power is defined as power generated at a wind speed of 11 m/s.
- Annual energy production is specified as power generated in a year assuming an average wind velocity of 5 m/s.

These rating values are shown below for the nominal turbine design:

\[ P_{\text{nom}} = 8 \text{ kW} \]
\[ AEP = 8,100 \text{ kWh} \]

The U.S. Energy Information Administration reports that the average residential energy consumption is 11,280 kWh per year, so the wind generated power from the design would account for about 70% of the total power consumed in a year.

**Wind Gusting Response**

A fixed speed wind turbine should operate at a constant shaft speed even when the wind velocity is changing in time. To test the Dymola model for this characteristic, a time dependent velocity input was connected to the turbine. The velocity input and rotor speed is shown in Figure 16.

![Figure 16: Velocity Gust Response (Rotor speed in red, Velocity in blue)](image)

As expected, the rotor speed is independent of the wind velocity. If closely scrutinized, the rotor speed does have a negligible variation from 22.62 rad/s to 22.65 rad/s during the velocity ramp, but this should not pose a problem in operation.
Design Variable Testing

The rotor radius is varied from 2.5 to 4.5 and the rated power is determined for each value. Figure 17 shows the results of this test.

![Figure 17: Rated Power vs. Rotor Radius](image)

The power seems to increase quadratically with the rotor radius, which matches the formulation in task 2 that showed the power increases with the square of the radius.

The tower height is varied from 20 m to 50 m in order to calculate the significance of tower height on power output. Figure 18 shows the results of this test.
Task 6 – Lessons Learned

- Creating a simulation with interaction between components is an iterative process. A component can appear to perform as expected when operating alone, but when attached to other components the behavior can change. It takes trial and error to troubleshoot and find the best solution.
- A good simulation strikes a balance between detail and ease of use. If more complexity is added to the model it may be more accurate but become nearly unusable. When creating a new model, it is a good idea to have this balance in mind.
- When testing a simulation, you must have in mind expected results based on calculation, references, or data, otherwise the results could be completely wrong. Many times users will believe the results of a simulation without properly testing and verifying that they can be accurate.
Task 7 – Project Webpage

A screenshot of the main webpage is shown in Figure 19.

Figure 19: Main Webpage View

Each link will lead to a figure or picture.
References


